MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

Edmund Dickson, F.G.S., 2 Starkie Street, Preston, Lancashire ;
Jacob Halm, Ph.D., Royal Observatory, Blackford Hill, Edinburgh ;
John William Hicks, Ordnance Committee Office, Woolwich ;
William Malin Hunt, 48–50, London Road, Nottingham ; and
Edwin Baikie Simpson-Baikie, Lieut. R.N.R., 142A The Bluff, Yokohama, Japan,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Arthur Grant Stillhamer, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A. (proposed by S. W. Burnham) ;
Ralph Falcon, M.A. Oxon., Barrister-at-Law, Camerton Hall, Workington, Cumberland (proposed by Cecil G. Dolmage).

Sixty-six presents were announced as having been received since the last meeting, including, amongst others :—

Publications of the Astronomical Laboratory, Groningen, Nos. 15, 16 (De Sitter, Photographic Parallax Tables, and

Kapteyn, Trigonometrical Formulæ), presented by the Laboratory ; G. W. Hill, Collected Mathematical Works, vol. iii., presented by the Author ; Oxford University Observatory, Miscellaneous Papers, vol. ii., presented by the Savilian Professor of Astronomy ; Radcliffe Catalogue of Stars for 1900 (A. A. Rambaut), presented by the Radcliffe Trustees ; Eighteen Charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich ; and Ten Charts presented by the San Fernando Observatory.

On the Ancient Eclipses of the Sun. By E. Nevill.

It was with much interest that I read Mr. Cowell's recent papers on the ancient eclipses of the Sun and the values they indicate for the secular accelerations of the longitudes of the Sun, Moon, and lunar node, and it is with some curiosity that I have waited for the last six months to see if any of the distinguished astronomers conversant with the subject would critically examine the results which Mr. Cowell has deduced.

In the first of these papers (*Monthly Notices*, vol. lxv. p. 861) Mr. Cowell reaches the conclusion that if the Moon's secular acceleration be taken as $+7''{\cdot}0$, in order to represent the ancient eclipses of the Sun it is necessary to reduce to about $+4''{\cdot}4$ the secular acceleration of the Moon's angular distance from its node, corresponding to a reduction of the secular acceleration of the Moon's node to only a third of its theoretical value.

In the second of these papers (*Monthly Notices*, vol. lxvi. p. 3) Mr. Cowell points out that, as this reduction of the Moon's nodical secular acceleration is inconsistent with the theory of gravitation, the requisite value of the secular acceleration of the Moon's angular distance from its node should be obtained by adopting the theoretical value of the secular acceleration of the node and increasing the secular acceleration of the Moon's mean longitude from $+6''{\cdot}8$ to $+10''{\cdot}9$, and ascribing a secular acceleration of $+4''{\cdot}1$ to the mean longitude of the Earth, so as to leave unchanged the epoch of conjunction of the Sun and Moon in mean longitude.

The results deduced in these two papers are quite distinct, and form two questions which may well be examined separately.

The first is the existence of an apparent secular acceleration of $+4''{\cdot}4$ in the angular distance of the Moon from its node.

The records of the ancient eclipses of the Sun and the values they indicate for the secular accelerations of the Moon's mean longitude, longitude of node, and longitude of perigee have been examined in some detail in one of the sections of my unpublished memoir on the errors of the lunar tables, and from it can be derived further information on the subject considered in Mr. Cowell's papers.

The eclipses are referred to Hansen's *Tables du Soleil* and to Hansen's *Tables de la Lune* under the following forms :—

A. Hansen's Tables, with his final values for the secular accelerations and motion of the node.

B. Hansen's Tables, after replacing his tabular values of the secular accelerations by those derived theoretically from the secular diminution of the eccentricity of the terrestrial orbit and making the small consequent corrections necessary to bring the amended tables into accord with the modern observations.

C. Hansen's Tables, after applying the corrections indicated by a complete discussion of the lunar observations between the years 1650 and 1900, employing for the secular acceleration of the mean longitude the value $+7''\cdot43$, of the longitude of the node the value $+6''\cdot07$, and of the longitude of the perigee the value $-32''\cdot53$.

The only eclipses considered are those at least two thousand years prior to the present epoch, as those of later date are of little real value for the desired purpose.

In the following $\Delta\phi$ represents the distance of the central line of totality from the assumed place of observation, measured on the meridian in degrees of latitude, a positive sign showing that the central line of totality is to the south of the assumed place of observation.

I. *The Eclipse of Agathocles.*—Recorded as having happened on -309 August 14, on the second day after Agathocles had left Syracuse on his way by sea towards Carthage. It being unknown whether he sailed south or north of Sicily, it is uncertain whether the eclipse was observed from $36\frac{1}{2}°$ or from 38° N. latitude, in both cases the longitude being about 15° E. The different modifications of the lunar tables yield the following values of $\Delta\phi$ measured from the latitude of Syracuse (37° N.), and to them has been now added the value that has been obtained from Mr. Cowell's data by an approximate calculation:

$$A = +1°\cdot8 \qquad B = +0°\cdot8 \qquad C = +0°\cdot8 \qquad \text{Cowell} = -0°\cdot9$$

As the width of the zone of totality exceeds two degrees, it is apparent that the eclipse is represented by all four systems of data.

II. *The Eclipse of Pelopidas.*—A very large, but not necessarily total, eclipse of the Sun, said by Plutarch to have occurred before Pelopidas left Thebes on the expedition against Alexander of Pheræ. Apparently it must have been the eclipse of -363 July 13. The values of $\Delta\phi$ are

$$A = -10°\cdot8 \qquad B = -11°\cdot8 \qquad C = -11°\cdot5 \qquad \text{Cowell} = -16°\cdot0$$

These all indicate a large eclipse, but the results are of little use.

III. *The Eclipse of Ennius.*—Stated by Cicero to have occurred at Rome in -399 June 21 toward sunset. It appears doubtful if the Sun was totally eclipsed before sunset, or even if

it were total at all. If there had been any certainty on this point it would have been a most valuable criterion, and deserved the great weight assigned it by Hansen. The values of $\Delta\phi$ are

$$A = -1^{\circ}\cdot2 \qquad B = +10^{\circ}\cdot4 \qquad C = +7^{\circ}\cdot4 \qquad \text{Cowell} = +3^{\circ}\cdot1$$

According to Hansen's values the Sun set after totality ; but according to the modified forms it must have set some time before totality, and the eclipse cannot have been total near Rome.

IV. *The Eclipse of Lysander.*—Stated by Xenophon to have occurred in the autumn of the year that Lysander took Athens, which would correspond to the eclipse of −403 September 3. It must have been a large eclipse, but not necessarily a total eclipse in Greece. The values of $\Delta\phi$ are

$$A = -0^{\circ}\cdot2 \qquad B = +2^{\circ}\cdot6 \qquad C = +1^{\circ}\cdot6 \qquad \text{Cowell} = -2^{\circ}\cdot4$$

The eclipse was annular, and according to all four sets of data must have been very large at Athens.

V. *The Eclipse of Thucydides.*—Stated by Thucydides to have occurred in the summer of the first year of the Peloponnesian War, and to have been nearly, but not quite, total at some spot in Greece—probably Athens, but quite possibly some place in the northern Ægean Sea, or even in the Ionian. It is this uncertainty which weakens the value of this eclipse as a test. The eclipse was only annular, and occurred so soon before sunset that stars might have been seen as recorded, even had the eclipse been only a large eclipse. The values of $\Delta\phi$ are

$$A = -17^{\circ}\cdot8 \qquad B = -1^{\circ}\cdot8 \qquad C = -5^{\circ}\cdot8 \qquad \text{Cowell} = +0^{\circ}\cdot1$$

Hansen's values fail to account for this eclipse ; both the theoretical secular accelerations and Mr. Cowell's data represent it as seen at Athens, and the data C as if seen from the northern Ægean.

VI. *The Eclipse of Thales.*—Stated by Herodotus to have occurred during a battle, fought probably at some place in Asia Minor, between the Lydians under Alyattes and the Medes under Cyaxares, at the end of a war of some length, during the period when Labynetus was king of Babylon. This war must have happened after the destruction of the Assyrian Empire by the Medes and Babylonians, and so have been later than the year −600. According to both Herodotus and Pliny, it was the eclipse predicted to the Ionians by Thales, and the date assigned by Pliny corresponds with that of −584 May 28.

There exist no means of ascertaining the position of the battlefield, and all that can be assured is that in all probability it was some spot on the great road from the Cilician Gates, through Iconium, to Sardis ; though there is always the possibility that it was on the northern road by which Cyrus subsequently invaded Lydia, which passed by Pteria, in Northern Cappadocia.

Airy, Hind, and Hansen identify this eclipse with that of −584 May 28, which, according to Hansen's Tables, crossed Southern Asia Minor; and it is difficult to reconcile the existing accounts with any other eclipse. It is true that recent investigation has shown that the data assumed in Hansen's Tables require such modification that the zone of totality of the eclipse of −609 September 28 may have passed over the north of Asia Minor, crossing the northern route afterwards followed by Cyrus; but as this eclipse happened before the fall of the Assyrian Empire it must have occurred prior to the war between the Medes and Lydians. Similar permissible changes in the data of Hansen's Tables show that the zone of totality of the eclipse of −556 May 17 may also have crossed the southern portion of Asia Minor, traversing the southern route from the Cilician Gates to Sardis, upon which the battle was fought in all probability; but if this eclipse be assumed, Crœsus could not have succeeded Alyattes until after −556, and it is difficult to compress the events of his reign into the ten years between that date and the capture of Sardis by Cyrus in about −545. Yet it is difficult to reconcile the earlier date of the eclipse with the statement of Herodotus that Labynetus was king of Babylon both at the epoch of the eclipse and at the capture of Sardis by Cyrus, as Nabu-kudur-uzur (Nabuchadrezzar II.), who reigned in Babylon in −584, died in −561; but if the later eclipse were adopted, then the king of Babylon at both events may have been Nabu-nahid (Nabonidus), who reigned from −556 to −537.

The values of $\Delta\phi$ for the eclipse of −584 May 28, measuring from Iconium, are

$$A = -2^{\circ}\cdot0 \qquad B = +3^{\circ}\cdot1 \qquad C = +1^{\circ}\cdot1 \qquad \text{Cowell} = -3^{\circ}\cdot0$$

and as the zone of totality considerably exceeded two degrees in width, all four sets of data will serve to bring the phase of totality within permissible limits of latitude. But according to the different data, totality is reached at sunset at east longitudes:

$$A = 44^{\circ} \qquad B = 23^{\circ} \qquad C = 29^{\circ} \qquad \text{Cowell} = 29^{\circ}$$

and as the probable longitude of the battlefield lies between 30° and 32°, except in the case of Hansen's data, the Sun must have set before the eclipse was total, though in all cases it would have been very largely eclipsed before reaching the western horizon.

Would this suffice to meet the exigencies of the account given by Herodotus?

The values of $\Delta\phi$ for the eclipse of −556 May 19 are

$$A = -5^{\circ}\cdot0 \qquad B = +2^{\circ}\cdot7 \qquad C = +1^{\circ}\cdot0 \qquad \text{Cowell} = +5^{\circ}\cdot0$$

and the zone of totality is narrow, little exceeding half a degree. The Sun in all cases would at totality be well above the western

horizon ; and according to data C, and possibly B, it would cross the probable position of the field of battle. Apart from chronological difficulties, this eclipse seems to agree with the account given by Herodotus even better than the earlier one. To throw over the circumstantial account given by Herodotus on the ground that he was credulous and related fables does not seem sound ; and it must be remembered that recent discoveries have generally confirmed the accuracy of Herodotus, even when his accounts seemed most improbable to earlier critics.

VII. *The Eclipse of Larissa.*—According to Xenophon this eclipse occurred at the capture of the "Median" city of Larissa by the "Persians" ; and, from the account of its effects, it must have been a total eclipse to have spread such consternation amongst the defenders of the city that they fled from the walls. The ruins of Larissa seen by Xenophon have been identified with certainty with the mounds of Nimrud, covering the remains of the Assyrian city of Calah. As the Sun represented Asshur, the ruling deity of Assyria, its eclipse would account for the terror of the defenders of Calah, who would take the darkness overspreading the Sun as indicating that the gods had decreed the ruin of Assyria.

With the imperfect knowledge of Assyrian chronology then alone available to guide him, Airy was led, in 1856, to identify this eclipse with that of −556 May 19, and showed that, according to Hansen's Tables, the narrow zone of totality passed nearly centrally over Larissa, and that there was no other eclipse within a period of forty years which could have been total at Larissa. But this eclipse cannot be reconciled with the theoretical value of the secular acceleration of the Moon's longitude, and it has proved the great obstacle to any reduction of Hansen's tabular values of the accelerations to the smaller values indicated by modern observations, as well as by theory, because any such reduction must throw the zone of totality of this eclipse many hundreds of miles to the south of Larissa.

Professor Newcomb and others seem disposed to evade the difficulty by either ignoring the eclipse, throwing over the tradition as unfounded, or ascribing the cause to some extraordinary meteorological phenomenon, and not to an eclipse of the Sun. I do not think this can be justified, and can see no reason for disbelieving the traditional account, which is so much in harmony with what is known of the Assyrian character ; all the more so as the whole course of recent discovery has tended to confirm the accuracy of these traditional records of the events of ancient history.

As the eclipse of −556 May 19 cannot be reconciled with the value of the secular acceleration indicated both by theory and by the modern observations of the Moon, it is certain that it cannot be the eclipse which occurred at the capture of Larissa.

Since the epoch of Airy's identification of this eclipse much progress has been made in our knowledge of Assyrian history

and chronology, and the actual dates of the greater part of the known history of Assyria have been determined with certainty; but, often as this eclipse has been discussed since Airy's memoir of 1857, it does not seem to have been noticed that the capture of Larissa, the Assyrian city of Calah, by either Medes or Persians cannot possibly have taken place in −556. Nineveh, Calah, and the other great cities of Assyria disappear from history before −600, and no reference to the existence of these great cities is known to occur at any subsequent period, either in Babylonian, Persian, Egyptian, or Grecian records. They were all destroyed together during the great invasion prior to −600, and the very state of the ruins at the present time shows that the destruction was sudden and once for all. Every inhabitant perished or was transported in slavery to some distant spot, and the cities were never allowed to be re-inhabited. The capture of Larissa to which the tradition refers must be the capture of the Assyrian Calah by the Medes and Babylonians prior to −600, and could not refer to the capture by the Persians of a town built by the Medes on the site of the older Assyrian city of Calah, a town of whose existence there is no record, and of whose remains there is no sign amongst the existing ruins of Calah. As before remarked, the very state of the ruins shows that Calah was an Assyrian city when destroyed prior to −600, and that its destruction was final.*

The great siege of Nineveh began in −608, and after a long siege ended with the destruction of the city in −605. Were the surrounding cities of Calah, Dur-Sargina, &c., besieged and captured before or after the capital, Nineveh? The whole tenor of ancient history shows that they would be besieged and taken one after the other, and not all at once. If Calah was besieged and taken before Nineveh, then the eclipse must have been that of −609 September 30. If besieged and captured after Nineveh had fallen, then the eclipse must have been that of −602 May 18.†

* Even if it had been permissible to suppose that the Medes, after capturing and destroying the city of Calah, had refounded a Median town on the site, though a supposition inconsistent both with history and with the state of the ruins, and that it was the capture by the Persians of this hypothetical new town of the Medes, to which the tradition referred, still even this would not save the eclipse of −556 May 19. For just as it is certain that the capture of the Assyrian city of Calah by the Medes and Babylonians occurred nearly half a century before this eclipse of −556 May 19, so is it equally certain that Cyrus, King of Ansan and Persia did not revolt against the Medes until the sixth year of Nabu-nahid (Nabonidus), King of Babylon, which corresponds with −548, eight years after the eclipse, and did not attack this region until the invasion of Babylonia in −538.

† It is known that the great palace of Calah was burnt down from some cause and rebuilt by Assur-bani-pal's successor, Assur-etil-ilani, some time between −620 and −615. It has been assumed that this burning took place during the Scythian (or Mandaian) invasion, which occurred about the time of Assur-bani-pal's death in −625. This assumed capture of Calah, and of Calah only, out of all the surrounding cities by the Scythians about −625 does not

The values of $\Delta\phi$ for these two eclipses are :

−602 May 18	A = +12°·0	B = +0°·2	C = +2°·0	Cowell = −0°·6
−609 Sept. 30	= − 9·7	= −1·2	= −3·4	= −8·9

The zone of totality of these eclipses was wide in each case, and both occurred near midday. Neither eclipse can have been total at Calah, according to Hansen's data ; but the eclipse of −602 May 18 was total on all three other systems of data, and the eclipse of −609 September 30 was total according to the theoretical values of the secular acceleration. This great obstacle to the adoption of the theoretical value of the secular accelerations vanishes.

VIII. *The Eclipse of Archilochus.*—Described by Archilochus in one of his poems, and from the description must have occurred at midday, and been at least annular, but probably total, where it was seen. Beyond this everything is vague. It may have been any eclipse of this nature between −710 and −640 ; and, though probably observed at Thasos, may have been seen from Paros, or even from some other place on the Ægean Sea. The eclipses during this period which it might have been are those occurring on

−710	March	14	Total
−703	October	19	Annular
−688	January	11	Annular
−656	April	15	Total
−650	June	7	Annular
−647	April	6	Total
−640	November	11	Annular

Mr. Cowell, like Professor Millosevich, selects the eclipse of −647 April 6 because it agrees with the data he adopts ; but this is considerably later than the date usually assigned to this

harmonise with the rest of the events of this period, nor even with probability. The very fact that an attempt was made to rebuild the palace, though in an inferior style, by Assur-bani-pal's successor, Assur-etil-ilani, would strongly indicate that the palace was destroyed by an accidental fire, similar to that which happened nearly three centuries before. If the city had been captured and burnt with the palace at this earlier epoch, it is not likely that an attempt would have been made to re-erect an inferior building amongst the ruins of a destroyed city when the king had so many other magnificent palaces in the surrounding cities. But if the palace of the second city of Assyria had been accidentally burnt, leaving the great city in existence, then it is probable enough that an attempt should have been made to restore the palace accidentally destroyed, though even in an inferior style, rather than leave the ancient capital of Assyria without any residence for the king. Certainly no attempt was made to restore any of the other palaces when once they had been destroyed with the surrounding cities during the conquest by the Medes and Babylonians.

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poet. Still, as pointed out by Mr. Lynn, Archilochus has referred to the capture of Magnesia by the Cimmerians, which took place during the reign of Assur-bani-pal of Assyria, and so must be later than —668, but probably before —660, the beginning of the trouble with Elam. The values of $\Delta\phi$ for these eclipses, supposed seen from Thasos, are :

—710 March 14	A = + 8°·1	B = − 10°·7	C = − 8°·6	Cowell =	...
—703 Oct. 19	− 7·3	+ 2·7	− 0·7		
—688 Jan. 11	+ 13·6	+ 6·3	+ 7·9		−1·8
—656 April 15	+ 16·1	− 2·0	+ 0·7		
—650 June 7	− 8·8	− 8·2	− 8·2		
—647 April 6	+ 0·3	− 12·9	− 9·6		+0·5
—640 Nov. 11	− 25·4	− 7·0	− 12·5		

From this it appears that the eclipse of Archilochus may be represented on the data A by assuming it to be the eclipse of —647 April 6, as seen from Thasos ; on the data B, employing the theoretical accelerations, by assuming it to be the eclipse of —656 April 15 as seen from Thasos or the eclipse of —703 October 19 as seen from Paros ; on the data C by assuming it to have been either the eclipse of —703 October 19 or that of —656 April 15, as seen from Thasos ; or by Mr. Cowell's data by supposing, as he does, that it was the eclipse of —647 April 6, as seen from Thasos, or the eclipse of —688 January 11.

It is apparent that this eclipse can afford no information as to the true values of the secular accelerations, as none of these seven eclipses, seen from either Thasos or Paros, can be found which will harmonise with any permissible values.

IX. *The Eclipse of Susa.*—Recorded in a tablet of the reign of Assur-bani-pal, immediately before his expedition against Elam in —659 : "Te-umman devised evil, and Sin devised against him forebodings of evil. In Tammuz an eclipse at evening. He troubled the lord of light, and the setting Sun thus also for three days was troubled. It went forth for the end of the reign of Elam." An eclipse of the Sun could not trouble the Sun for three days, but Dr. Pinches considers that the eclipse and the troubling of the Sun should be regarded as distinct occurrences. Even were there objections to so differencing them, the record may be interpreted as reading that the Sun was so troubled or eclipsed "*on*" three days, or that there had been three eclipses, probably at sunset.

The inscription does not say that the eclipse was total ; a large eclipse at sunset would suffice ; but the month, Tammuz, of the specified eclipse is only consistent with the eclipse of —660 June 27. The values of $\Delta\phi$ are :

A = +1°·6 B = +13°·5 C = +7°·9 Cowell = +6°·0

All occurred near sunset. According to Hansen's data it would have been annular before sunset ; but, according to the data C

and those adopted by Mr. Cowell, only a large partial eclipse at sunset, whilst the system depending on the purely theoretical data would make it a much smaller eclipse. There are two other eclipses at this time ; one, that of −661 January 12, would have been a large eclipse at sunset ; and another, in −663, would also have been visible, but not at sunset.

X. *The Eclipse of Esar-haddon.*—Recorded in the form, "Since my lord went to Egypt an eclipse has taken place in the month of Tammuz" (June), on an Assyrian tablet of the reign of Esar-haddon, between −680 and −668. The only large eclipse which can answer to this description is that which occurred on −678 June 17. There is absolutely no other in the reign of this king which it could have been. Yet this eclipse occurred several years before Esar-haddon's great Egyptian wars ; and, if it be the eclipse of this date, the statement that it occurred when the king had gone to Egypt must refer to one of the early expeditions to the frontiers of the empire, which are only imperfectly recorded in the early annals of this king of Assyria.

The values of $\Delta\phi$ are :

A = +19°·6 B = +5°·6 C = +11°·3 Cowell = +4°·8

It will be seen that this eclipse would have been a very large eclipse at Nineveh according to the data employed by Mr. Cowell, or those depending on the theoretical secular accelerations.

XI. *The Eclipse of Nineveh.*—Recorded under the form "Eponomy of Bur-Sagall, Governor of Gozan. A revolt in Assur took place in the month Sivan, and the Sun was eclipsed" on an Assyrian cuneiform tablet as occurring in the eighth year of Assur-dan III., corresponding to −762. Hence the eclipse must have been that of −762 June 15. It will be seen that the eclipse is not stated to have been total, nor is anything recorded indicating that it was total ; and though undoubtedly it must have been large, it must not be assumed to be necessarily a total eclipse, as assumed by Mr. Cowell. In consequence it is not of great value for ascertaining the probable amount of the secular accelerations, though highly important as fixing Assyrian chronology. The values of $\Delta\phi$ are :

A = −4°·1 B = −6°·7 C = −5°·9 Cowell = −0°·8

As the zone of totality was fairly wide, Mr. Cowell's data make it total at Nineveh, but on the other systems it is only very large. Either would satisfy the record.

These eclipses are considered in my memoir, and to them may be added the new eclipse recorded in the Babylonian tablet deciphered by Mr. King and quoted in part by Mr. Cowell.

XII. *The Eclipse of Babylon.*—Recorded as occurring on the "26th day of the month Sivan in the 7th year." From the other details the eclipse was most likely a total eclipse with a vivid

corona. Assuming that the eclipse occurred in the first half of the eleventh century it would seem certain that this eclipse must have been that of −1069 June 19, which occurred in the month Sivan of the seventh year of E-Ulmas-sakin-sumi, King of Babylon, thus being in strict accord with the record. But as this eclipse could not possibly have been total at Babylon according to his adopted data, Mr. Cowell is led to substitute the eclipse of −1062 July 30. The discrepancy of seven years in the epoch of the king's reign might not be deemed fatal to this identification, owing to the uncertainty in these very early epochs; but can July 30 ever have been in the month Sivan, which corresponds with May or June? Could it even have corresponded with Tammuz, the next month, or must it not be held to be some day of the month after? As the inscription must be regarded as an actual record, the month cannot have been in error, and the record cannot be changed by reading Tammuz, or even some later month, for the Sivan actually recorded.

This eclipse of −1069 June 19 is the only one which occurred during the month Sivan which could have been visible as a total eclipse from Babylon during the first half of the eleventh century. There are annular eclipses in this month during this period, but they could not have been even large eclipses near Babylon, and there are other eclipses which might have been total near Babylon, but could never have been described as occurring in the month Sivan.

If it were permissible to assume an earlier epoch, then the record might refer to one of the kings of the Isin dynasty, and correspond with either the eclipse of −1116 June 28 or −1123 May 18, both of which may have been total at Babylon.

The values of $\Delta\phi$ for the preceding eclipses are:

-1062 July 30	A = − 2°·2	B = − 7°·0	C = − 5°·6	Cowell = + 0°·8
−1069 June 19	= +30·7	= +13·0	= +16·5	= +12·3
−1116 June 28	= +10·8	= − 6·9	= − 0·3	= + 4·0
−1123 May 18	= +11·7	= + 1·2	= + 3·2	= − 2·1

It will be seen that the line of totality of the eclipse of −1069 June 19 cannot be brought over Babylon by any of the four systems of data or by any system within the limits of possibility. Hence, though otherwise in complete accordance with the recorded account, it cannot have been the recorded eclipse. The eclipse of −1062 July 20 can be made total at Babylon either by Mr. Cowell's data or by a slight modification of the very different data obtained by Hansen, and therefore by any number of intermediate combinations lying between the two systems. If it be adopted as the eclipse corresponding to that recorded in the cuneiform tablet, it must be held as of little use for indicating the true value of the secular acceleration; but the discordance between the month of the eclipse and that stated in the inscription must be held to render such an identification very doubtful.

Mr. Cowell does not give any reasons for assigning this eclipse to a period subsequent to −1100, nor does he state whether Mr. King furnished such reasons, nor even whether the rest of the inscription (by stating the king's name or dynasty) affords means for fixing such a limit. Unless such reasons exist for dating the record after −1100, the eclipse would seem more likely to have been that of −1116 June 28, or −1123 May 18, than that of −1062 July 30, since the eclipse of −1069 June 19 is out of the question, as not even large near Babylon. If the inscription permits of the recorded eclipse being identified with one of these earlier eclipses it would be most valuable for fixing the true value of the secular acceleration and motion of the lunar node, as it would clearly distinguish between the four representative sets of data.

The preceding comparisons show that the eclipses, which by slight permissible alterations in the assumed data may be represented by the four very different sets of assumed values for the secular accelerations and motions of the Moon's node, are as follows:—

Name.	Obs. Phenomenon.	A		B		C		Cowell.	
		Date.	Δφ.	Date.	Δφ.	Date.	Δφ.	Date.	Δφ.
I. Eclipse of Agathocles	Total	− 309	+1°·8	− 309	+0°·8	− 309	+0°·8	− 309	−0°
III. Eclipse of Ennius ...	Large	− 399	−1·2	...	...	...	...	− 399	+3°
IV. Eclipse of Lysander	Large	− 403	−0·2	− 403	+2·6	− 403	+1·6	− 403	−2°
V. Eclipse of Thucydides	Nearly total	...	...	− 430	−1·8	− 430	−2·8*	− 430	+0°
VI. Eclipse of Thales ...	Total	− 584	−2·0	...	...	− 556	+1·0	...	...
						− 584	+1·1	...	...
VII. Eclipse of Larissa ...	Total	− 556?	−0·6	− 602	+0·2	− 602	+2·0	− 602	−0°
VIII. Eclipse of Archilochus	Total	− 647	+0·3	− 656	−2·0	− 656	−0·7	− 647	+0°
				− 703	+2·7	− 703	+0·7	− 688	−1°
IX. Eclipse of Susa ...	Large	− 660	+1·6	...	...	− 660	+7·9	− 660	+6°
X. Eclipse of Esarhaddon	Large	...	...	− 678	+5·6	...	...	− 678	+4°
XI. Eclipse of Nineveh...	Large	− 762	−4·1	− 762	−6·7	− 762	−5·9	− 762	−0°
XII. Eclipse of Babylon	Total	−1062	−2·2	−1123	+1·2	−1116	−0·3	−1062	+0°
								−1123	−2°

Mr. Cowell considers that the fact that the system of secular mean motions and accelerations adopted by him renders total the five eclipses

−1062	July	30	at Babylon
− 762	June	14	,, Nineveh
− 647	April	5	,, Thasos
− 430	August	3	,, Athens
+ 197	June	3	,, Utica

* Observed from the northern Ægean instead of Athens.

is so remarkable that it cannot be held a chance coincidence, but must be regarded as demonstrating the accuracy of the adopted system. It is necessary to remember, however, in considering this claim—

That the eclipse at Babylon is stated explicitly to have occurred in the month Sivan, whereas the eclipse adopted occurred in Tammuz or later ;

that the eclipse at Nineveh is not recorded as being total, as assumed without authority by Mr. Cowell, and so may have been only a large eclipse ;

that a total eclipse may be found to represent that at Thasos on any system of data, so that it is valueless as evidence of the accuracy of any system ;

that the eclipse at Athens, though annular according to Mr. Cowell's data, is only recorded as having been very large, whilst it is represented just as well by other systems of data as by that adopted by him ; and

that the eclipse of Utica has not been considered, as its epoch is so late that it cannot be regarded as of any value.

Taking these considerations into account it will be seen that the fact that these five eclipses selected by Mr. Cowell are rendered total by his data cannot be regarded as establishing the accuracy of the data adopted. Nor can it be urged that, though each of the preceding five eclipses may be represented just as well by other systems of data, yet Mr. Cowell's is the only system which will represent five eclipses, for the preceding examination shows that out of the twelve eclipses considered Hansen's theoretical values can be made to represent seven or possibly nine ; that the strictly theoretical values of the secular accelerations will represent seven or eight; that the data derived exclusively from the modern observations will represent six at least, but possibly eight; and that Mr. Cowell's data with some slight amendment may be made to represent as many as ten eclipses. The mere fact that a system of data represents a certain number of rather vaguely defined eclipses cannot be regarded as establishing anything.

In truth, the results which have been given show that unless it can be established that an eclipse was to all intents total at a specified place within a narrow definite period of years, it can be of small use as a criterion of the true values of the mean motions and secular accelerations. Of those in the preceding list there is but one possessing these requirements, and that is the eclipse of Larissa, though possibly the eclipse of Babylon may prove to be another.

In my memoir, in addition to the series of eclipses which has been considered in the preceding, there was also taken into account a series of twelve eclipses of the Sun, observed during the Chow dynasty in China, and presumably observed from the

ancient capital of Heeng-Yang, in Shensee, where there still exists the remains of a very ancient observatory. It is possible, however, that in some cases the place of observation may have been the neighbouring city of Chang-gan, which was the residence of some of the emperors during this period. Some of these eclipses are expressly recorded as total; and all seem to have been very large eclipses, if not at least annular. In each case the exact date of the eclipse is definitely fixed by the record, so that there is no uncertainty in this respect.

The eclipses are :—

			A	B	C	Cowell.
			°	°	°	°
XIII.	−497 Sept. 22	Annular $\Delta\phi$ =	− 4·2	= + 3·6	= + 1·8	= − 4·2
XIV.	−526 April 18	Annular	+ 8·3	− 3·6	+ 0·0	− 5·4
XV.	−545 Oct. 3	Total	− 6·0	+ 4·2	+ 2·9	+ 6·8
XVI.	−548 June 19	Total	+ 4·2	+ 3·4	+ 3·3	− 0·2
XVII.	−549 Jan. 5	Annular	+ 13·2	+ 1·5	+ 6·4	+ 11·7
XVIII.	−574 May 9	Total	− 1·1	− 1·9	− 1·4	+ 2·2
XIX.	−600 Sept. 20	Total	− 20·1	− 2·3	− 5·1	+ 2·0
XX.	−625 Feb. 3	Annular	+ 6·9	− 0·3	+ 2·4	+ 8·6
XXI.	−663 Aug. 28	Total	− 14·8	− 3·4	− 5·4	− 12·2
XXII.	−667 Nov. 10	Annular	− 12·3	− 0·9	− 4·7	− 11·7
XXIII.	−668 May 27	Annular	− 3·5	− 7·3	− 5·5	+ 0·3
XXIV.	−708 July 17	Total	− 19·9	− 4·2	− 5·8	− 3·0

There are several other eclipses during this period recorded as having occurred, whose place of observation is too uncertain to render them of value.

Use was also made of four more recent eclipses, recorded in more indefinite language, only the year and season being stated, but all explicitly described as total. These are the eclipses :—

			°	°	°	°
XXV.	−180 Mar. 4	Total $\Delta\phi$ =	+ 1·6	= − 3·5	= − 2·1	= + 0·9
XXVI.	−187 July 17	Total	− 8·3	+ 1·1	− 1·0	+ 1·9
XXVII.	−299 July 26	Total	− 2·9	− 6·9	− 5·9	− 3·3
XXVIII.	−381 July 13	Total	− 6·0	− 13·8	− 11·2	− 15·6

This last eclipse is discordant on all systems of data, and could only have been total in Mongolia.

It will be seen that out of these sixteen eclipses observed in China, by regarding those for which $\Delta\phi$ is less than $2\frac{1}{2}°$ as total or annular, and those for which $\Delta\phi$ lies between $2\frac{1}{2}$ and 6° as so large and conspicuous as to be practically total, the number in the case of each of the four sets of data being considered is

	A	B	C	Cowell.
Total or annular	2	6	6	6
Very large	5	7	8	4
Discordant	5	1	1	4

Practically the last three systems all serve to represent these Chinese eclipses as well as could be anticipated, and show the caution that must be employed in drawing deductions from such a circumstance.

In concluding this examination it must be pointed out that the preceding considerations do not invalidate Mr. Cowell's conclusion that the Moon's argument of latitude F possesses a secular acceleration of about $+4''\cdot4$ per century; they show that the eclipses adduced by Mr. Cowell do not suffice to establish it as he supposed, and are quite consistent with systems of data very different from that adopted by him, and which do not involve the difficulties inseparable from his. But it must be distinctly understood that this is all. In many respects the preceding examination of the ancient eclipses strengthens Mr. Cowell's view, for it shows that his data serve to represent other eclipses besides those considered by him, and in particular that they serve to represent the eclipse of Larissa, the most critically important of them all. Indications of the existence of such a secular term, though smaller in amount, have been found cropping up repeatedly in the discussion of the early observations; but it is very difficult to differentiate such a secular term from the very similar effects which would be produced by inequalities of very long period in the complete expression for the Moon's latitude, and there is some reason for supposing these to exist.

A much wider question is raised by Mr. Cowell in his second paper, that the apparent secular acceleration of $+4''\cdot4$ in the angular distance of the Moon from its node really arises from the existence of a secular acceleration of $+4''\cdot1$ in the mean longitude of the Earth.

Assuming that the ancient eclipses of the Sun show that there exist a secular acceleration of $+4''\cdot4$ in the angular distance of the Moon from the node and a secular acceleration of $+6''\cdot8$ in the angular distance of the Moon from the Sun, Mr. Cowell points out that this can be represented by supposing the secular acceleration of the Moon's node to have the theoretical value of $+6''\cdot5$, and assigning a secular acceleration of $6''\cdot8+4''\cdot1=10''\cdot9$ to the mean longitude of the Moon, and of $+4''\cdot1$ to the apparent mean longitude of the Sun.

The effect of this change of origin of the secular acceleration leaves unchanged the epoch of conjunction of the mean longitudes of the Sun and Moon, and does not alter the values of either $D=(L-L')$ or $F=(L-\Omega)$; hence Mr. Cowell concludes that it leaves unchanged the figures of his eclipse calculations, except perhaps a small change of less than $20''$ in the correction for parallax, which he regards as quite unimportant.*

* This is an oversight of Mr. Cowell's. It is true that neither D nor F is altered, but both g and g' are changed by quantities which may exceed a degree of arc. Hence both V and V′ and their difference V−V′ may be changed by considerable quantities, possibly amounting to several hundreds of seconds of arc in the case of the earlier eclipses; whilst U will also be

But it will be seen that this new hypothesis as to the origin of the correction required by the Moon's argument of latitude removes one theoretical difficulty only at the expense of restoring another; for though the assumed secular acceleration of the Moon's node is brought into accord with theory, it is only at the cost of reinstating the discordance between 6″·0, the theoretical value of the secular acceleration in mean longitude, and that of 10″·9, the value necessitated by Mr. Cowell's new hypothesis. It is true that such a discordance conceivably might arise from tides in the body of the Earth, regarded as a thin rigid shell covering a viscous fluid interior, or in an ocean entirely covering the surface of the Earth, though not from tides in smaller oceans having the physical configuration of those actually existing on the terrestrial surface; but there is no real evidence in support of a tidal retardation in the rotation of the Earth of a nature which could be held to account for a material discordance between the observed value of the secular acceleration of the Moon's mean longitude and that derived by theory from the diminution in the eccentricity of the Earth's orbit.

The effect of this amended hypothesis for bringing the lunar tables into accord with the ancient eclipses by assuming the existence of a considerable secular acceleration in the motion of the Earth from the effects of a resisting medium in space proceeds far beyond the effect on the eclipses, but, except in respect of its effects on the observations of the transits of *Mercury*, these have not been considered by Mr. Cowell. It is not proposed, however, to discuss the matter beyond its effects on the apparent longitudes of the Sun and Moon, though these form a very small portion of the difficulties which would have to be overcome in reconciling with observation the effects due to this new hypothesis of Mr. Cowell's. The assumed secular retardation cannot be supposed to be confined merely to the Earth; so the assumed law of resistance in space must be such that, despite the great physical differences between the different planets, the apparent secular accelerations of the other planets would differ so slightly from the apparent secular acceleration of the Earth that no appreciable signs of the difference would be apparent. The more important cases would be those of *Venus* and *Mars*, whose observed secular variations are closely in accord with those derived from theory, and Mr. Cowell would have some difficulty in finding a law of resistance in space which would leave the relative motions of the three planets unaffected. The effects of the resisting medium would extend, of course, to the eccentricities as well as to the mean distances.

altered, though to a smaller degree. Hence Mr. Cowell's eclipse calculations will no longer apply, and must be revised to meet the effects of the changed hypotheses. In the case of the five particular eclipses considered by Mr. Cowell it is probable that small alterations in his data will suffice to bring them into accord with the changed conditions, and so leave his conclusions unaffected.

Restricting the consideration to the effects on the mean longitudes of the Sun and Moon, assuming 1900 as the epoch, the effect of the new secular term would be to increase the tabular longitudes by a second of arc in 1850, so it will be necessary to increase the secular mean motions by some two seconds of arc, in order to bring the new tabular places into the same accord with observation as exists with the present tables as revised. The effect will be to increase the tabular longitudes of the Sun and Moon at different epochs by the quantities given in column I. of the table which follows. But it will be seen that this leaves discordances of over 6″ between Bradley's observations of 1750–60 and the tables as altered, discordances far greater than are admissible; hence to bring these earlier observations into accord with the changed tables a still further increase must be made in the secular mean motion, with the consequence that if the ancient eclipses are still to be represented, a further increase must also be made in the value of the new secular acceleration. Adopting +5″·5 for the centennial increase in the mean longitude and +4″·33 for the coefficient of the new secular term, the tabular places of the Sun and Moon at the different epochs will be changed by the quantities shown in Column II. For comparison, the columns headed "Sun" and "Moon" show the outstanding discordances between the observed and latest revised tabular places of the Sun and Moon, and the columns "Sun+II." and "Moon+II." the residuals which will remain outstanding when compared with tables modified by the quantities given under Column II.

Epoch.	I.	II.	Sun.	Sun+II.	Moon.	Moon+II.
	″	″	″	″	″	″
1900	+ ·00	+ ·00	+ ·20	+ ·20	+ ·55	+ ·55
1880	− ·24	− ·93	− ·07	−1·00	+ ·12	− ·81
1860	− ·14	− 1·51	− ·41	−1·92	− ·04	− 1·55
1840	+ ·28	− 1·75	− ·45	−2·20	− ·21	− 1·96
1820	+ 1·02	− 1·63	+1·11	− ·52	+ ·52	− 1·11
1800	+ 2·10	− 1·17	− ·72	−1·89	− ·75	− 1·92
1780	+ 3·50	− ·37	− ·95	−1·32	+ ·79	+ ·42
1760	+ 5·24	+ ·78	+ ·22	+1·00	−1·70	− ·92
1740	+ 7·30	+ 2·28	+ ·30	+2·58	−1·94	+ ·34
1720	+ 9·68	+ 4·12	...	...	− ·13	+ 3·99
1700	+12·44	+ 6·65	...	...	− ·39	+ 6·26
1680	+15·44	+ 8·85	...	...	+1·91	+10·74
1650	+18·81	+11·74	...	...	+ ·98	+12·72

The resulting discordances between the tabular and observed places of the Moon between 1740 and 1900 could be eliminated by an increase in the assumed coefficients of the terms of very long period arising from the perturbations by the planets, but it

is doubtful if the very large discordances between the observed and tabular times of the numerous occultations of stars observed between 1680 and 1730 could be similarly eliminated. But there is no such means of removing the discordances which the change would introduce between the observed and tabular places of the Sun during the period 1750–1880, and these discordances are far greater than can be ascribed to the outstanding errors of observation.

Natal Observatory:
1906 *April* 3.

On the Ratios of the triangles in the Determination of the Elliptic Orbit from three Observations. By Shin Hirayama.

1. In this paper I determine the ratios of the triangles which play an important part in the problem of determining the orbit from three observations. Gibbs has determined these ratios by expanding the heliocentric rectangular coordinates of the planet in powers of time, while I expand them in a series of sines and cosines of the mean anomaly and in powers of eccentricity, as is generally done in the elliptic theory.

2. For the three observations of the planet at the times t_1, t_2, and t_3 we have :

$$t_3-t_2=\tau_1. \qquad t_3-t_1=\tau_2, \qquad t_2-t_1=\tau_3.$$

x_1, y_1, z_1
x_2, y_2, z_2 } three rectangular heliocentric coordinates of planet.
x_3, y_3, z_3

r_1, r_2, r_3 radii vectores of planet.

n mean daily motion expressed in seconds of arc.

a semi-major axis of planet.

3. Let

$$x=a_0+a_1\sin nt+b_1\cos nt+a_2\sin 2nt+b_2\cos 2nt \quad . \quad . \quad (1)$$

where a_0, a_1, b_1, a_2, b_2, are five constants depending on the elements of the orbit. Then the accelerations along the x axis corresponding to the three positions will be represented by the following expressions respectively :

For $t=-\tau_3$; $\dfrac{d^2x}{dt^2}=-\dfrac{x_1}{{r_1}^3}=-n^2a_1\sin n\tau_3+n^2b_1\cos n\tau_3-4n^2a_2\sin 2n\tau_3+4n^2b_2\cos 2$

For $t=0$; $\dfrac{d^2x}{dt^2}=-\dfrac{x_2}{{r_2}^3}=$ n^2b_1 $+4n^2b_2$

For $t=+\tau_1$; $\dfrac{d^2x}{dt^2}=-\dfrac{x_3}{{r_3}^3}=+n^2a_1\sin n\tau_1+n^2b_1\cos n\tau_1+4n^2a_2\sin 2n\tau_1+4n^2b_2\cos 2$

Hence we have

$$\begin{aligned}
x_1 &= a_0 - a_1 \sin n\tau_3 + b_1 \cos n\tau_3 - a_2 \sin 2n\tau_3 + b_2 \cos 2n\tau_3 \\
x_2 &= a_0 \qquad\qquad + b_1 \qquad\qquad + b_2 \\
x_3 &= a_0 + a_1 \sin n\tau_1 + b_1 \cos n\tau_1 + a_2 \sin 2n\tau_1 + b_2 \cos 2n\tau_1 \\
\frac{x_1}{n^2 r_1^3} &= -a_1 \sin n\tau_3 + b_1 \cos n\tau_3 - 4a_2 \sin 2n\tau_3 + 4b_2 \cos 2n\tau_3 \\
\frac{x_2}{n^2 r_2^3} &= \qquad + b_1 \qquad\qquad + 4b_2 \\
\frac{x_3}{n^2 r_3^3} &= +a_1 \sin n\tau_1 + b_1 \cos n\tau_1 + 4a_2 \sin 2n\tau_1 + 4b_2 \cos 2n\tau_1
\end{aligned}$$

From these six equations the five constants a_0, a_1, b_1, a_2, b_2 may be eliminated, leaving an equation of the form

$$\left.\begin{aligned}
&\left[4(S_2 \sin n\tau_1 - S_1 \sin 2n\tau_1) + \frac{4S_1 \sin 2n\tau_1 - S_2 \sin n\tau_1}{n^2 r_1^3}\right] x_1 \\
&+\left[4(S_2 \sin n\tau_2 - S_1 \sin 2n\tau_2) + \frac{4S_1 \sin 2n\tau_2 - S_2 \sin n\tau_2}{n^2 r_2^3}\right] x_2 \\
&+\left[4(S_3 \sin n\tau_3 - S_1 \sin 2n\tau_3) + \frac{4S_1 \sin 2n\tau_3 - S_2 \sin n\tau_3}{n^2 r_3^3}\right] x_3 = 0
\end{aligned}\right\} \ldots (2)$$

where

$$\begin{aligned}
S_1 &= \sin n\tau_1 + \sin n\tau_2 + \sin n\tau_3 = -4 \sin \frac{n\tau_1}{2} \sin \frac{n\tau_2}{2} \sin \frac{n\tau_3}{2} \\
S_2 &= \sin 2n\tau_1 + \sin 2n\tau_2 + \sin 2n\tau_3 = -4 \sin n\tau_1 \sin n\tau_2 \sin n\tau_3
\end{aligned}$$

If we put

$$\left.\begin{aligned}
S &= \cos \frac{n\tau_1}{2} \cos \frac{n\tau_2}{2} \cos \frac{n\tau_3}{2} \\
P_1 &= 4S - \cos n\tau_1 = 1 + \cos n\tau_2 + \cos n\tau_3 \\
P_2 &= 4S - \cos n\tau_2 = 1 + \cos n\tau_3 + \cos n\tau_1 \\
P_3 &= 4S - \cos n\tau_3 = 1 + \cos n\tau_1 + \cos n\tau_2 \\
Q_1 &= \cos n\tau_1 - S = 3S - P_1 \\
Q_2 &= -\cos n\tau_2 + S = -3S + P_2 \\
Q_3 &= \cos n\tau_3 - S = 3S - P_3
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (3)$$

then we easily derive from (2) the following equation:

$$\begin{aligned}
\sin n\tau_1 \left(P_1 + \frac{Q_1}{n^2 r_1^3}\right) x_1 - \sin n\tau_2 \left(P_2 - \frac{Q_2}{n^2 r_2^3}\right) x_2 \\
+ \sin n\tau_3 \left(P_3 + \frac{Q_3}{n^2 r_3^3}\right) x_3 = 0 \qquad \ldots (4)
\end{aligned}$$

Whence we see that the three triangles determined each by a pair of the three radii vectores, and usually denoted by $[r_2r_3]$, $[r_1r_3]$, $[r_1r_2]$, are respectively proportional to

$$\left(P_1+\frac{Q_1}{n^2r_1^3}\right)\sin n\tau_1,\quad \left(P_2-\frac{Q_2}{n^2r_2^3}\right)\sin n\tau_2,\quad \left(P_3+\frac{Q_3}{n^2r_3^3}\right)\sin n\tau_3 \qquad (5)$$

4. Expand S, P_1, Q_1, &c. in powers of τ, and neglect the higher powers of τ. Substituting all these values in (4), and after dividing by common factors, we get the equation

$$\tau_1\left(1+\frac{\tau_2\tau_3-\tau_1^2}{12r_1^3}\right)x_1-\tau_2\left(1-\frac{\tau_1\tau_3+\tau_2^2}{12r_2^3}\right)x_2+\tau_3\left(1+\frac{\tau_1\tau_2-\tau_3^2}{12r_3^3}\right)x_3=0$$

which is the equation first obtained by Gibbs.

5. In the above explained method for the determination of the ratios of triangles we assume a suitable value of n, and compute these ratios by (3) and (5). We find thus a very close approximation to their correct values if the eccentricity of the ellipse be small. The starting equation (1) shows that there we have already neglected certain terms of the squares of eccentricity.

6. To test the accuracy of the preceding formulæ let us take the *Ceres* example treated by Gauss. The ratios of triangles are computed for four different values of n; while the rigorous values of $[r_2r_3]$, $[r_1r_3]$, and $[r_1r_2]$ are computed by the expressions $r_2r_3\sin(v_3-v_2)$, $r_1r_3\sin(v_3-v_1)$, and $r_1r_2\sin(v_2-v_1)$ respectively. The results are shown in the following table, the corresponding values from Oppolzer and Gibbs being added for comparison.

		$\log\frac{[r_2r_3]}{[r_1r_3]}$.	Error.	$\log\frac{[r_1r_2]}{[r_1r_3]}$.	Error.
Rigorous Formula		9·751242	0	9·776875	0
Oppolzer's „		9·742730	+8512	9·768466	+8409
Gibbs' „		9·751316	− 74	9·776953	− 78
Hirayama's Formula $n=$	1000″	9·751122	+ 120	9·777070	− 95
	769·6845	9·751280	+ 38	9·776859	+ 16
	750	9·751287	− 45	9·776853	+ 22
	500	9·751323	− 81	9·776864	+ 9

Astronomical Observatory, Azabu, Tōkyō:
1906 *March* 19.

On the Orbit and Relative Masses of β 733 (85 Pegasi).

By W. Bowyer and H. H. Furner.

β 733.

	h m s	
R.A.	23 56 57	1900 Mags. 6 : 10
N.P.D.	63° 27′	

The discussion of the meridian observations of 85 *Pegasi* for a determination of the relative masses, incidentally showed that the period of this binary did not exceed twenty-five years.

The orbit has been computed at various times, the results so far as the period is concerned being—

	Years.
1888 Schaeberle...	22·3
1892 Glasenapp	17·5
1895 See...	24·0
1899 Burnham	25·7

This failure to secure harmony has led to a discussion of the micrometer measures, which are best satisfied by a period of twenty-six years—*i.e.* a little larger than that deduced by Professor Burnham.

The meridian observations extend over a period of seventy-five years, or three revolutions, and the period given can be assumed to be more accurate than that at present derived from the micrometer measures. One explanation of the discrepancy may be found in the difficulty of measuring so close and dissimilar a pair ; but there is a tendency to periods of alternate positive and negative residuals when comparing observed and computed places which would point to some real disturbing cause. This would show to a greater extent in the micrometer measures than in the meridian observations.

In Fig. 1 all available measures have been taken in groups, plotted down, and the apparent ellipse drawn. From this the elements of the true ellipse have been found to be :—

$e = ·46$	$a = 0''·82$
$\Omega = 115^\circ\ 38'$	$T = 1883·5$
$\gamma = 53\ 5$	$P = 26·3$ years
$\lambda = 266\ 7$	

From the apparent ellipse the relative movement of the two components is found to be $1''·35$ in right ascension and $1''·05$ in

north polar distance, these being the values used in the following discussion of the masses.

Several more years' micrometer measures would appear desirable before a really satisfactory investigation of the system can be made. Of late, observations of the pair are somewhat scarce, but double-star observers would do well to keep this binary under careful measurement for some time to come.

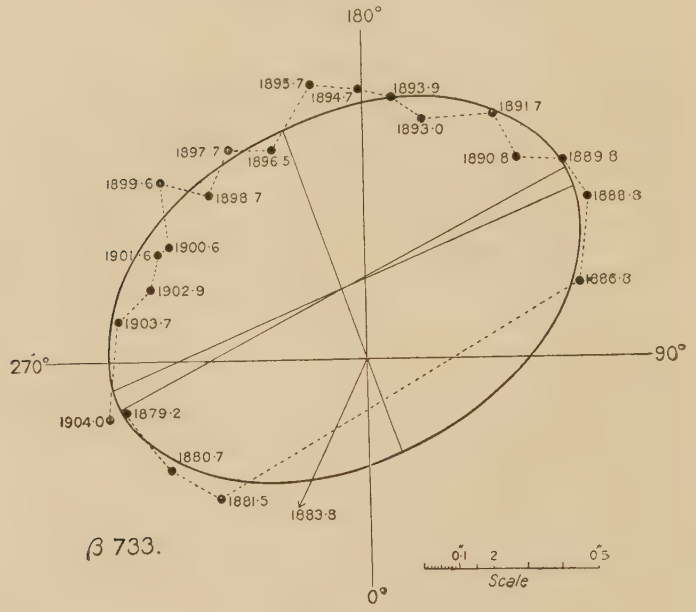

Fig. 1.—Apparent Ellipse.

On the Mass of 85 *Pegasi.*

The system 85 *Pegasi* consists of a sixth-magnitude star whose colour is yellow, and a tenth-magnitude star whose colour is blue. It is a close pair, but the great difference in brightness allows us to assume that the meridian observations are of the bright component, or that the effect of the faint companion on the bisections may be neglected. The following observations have been corrected by Auwers' corrections and for proper motion.

The observations extend over a period of seventy-five years, and so actually date two complete revolutions anterior to the discovery of the faint companion by Burnham in 1878. A very good determination of the period is therefore possible. It may be taken as 24·5 years. The meridian observations give a range

of 1″·15 in R.A., and 0″·8 in N.P.D. (see fig. 2), corresponding to a range of 1″·35 and 1″·05 in the relative orbit, and hence

$$\frac{\text{Mass A}}{\text{Mass B}} = \frac{1\cdot35-1\cdot15}{1\cdot15} = \frac{4}{23} \text{ from R.A.}$$

and

$$= \frac{1\cdot05-0\cdot8}{\cdot8} = \frac{5}{16} \text{ from N.P.D.}$$

giving equal weight to each, the mass of the faint star is approximately four times that of the bright star.

Right Ascensions and North Polar Distances of 85 Pegasi reduced to the Epoch 1900·0 with Auwers' Systematic Corrections applied. Adopted Proper Motions.

$+0^{s}\cdot0612$ *in R.A. and* $+0''\cdot980$ *in N.P.D.*

Cat.	Epoch of Obs. R.A.	No. of Obs.	Secs. R.A. 1900·0.	Epoch of Obs. N.P.D.	No. of Obs.	Secs. N.P.D. 1900·0.
			s			″
Argelander ...	1830	11	56·77	1830	11	50·3
Armagh ...	1831·6	6	56·70	1846·2	7	48·1
Taylor ...	1835·4	10	56·72	1836·5	10	50·2
Rumker ...	1845·0	3	56·67	1845·0	3	48·4
Paris	1849·8	2	56·82	...	...	...
Radcliffe ...	1857·3	4	56·76	1860·1	3	51·8
Brussels ...	1860·8	25	56·71	1857·5	18	50·4
Paris	1864·1	21	56·74	1861·8	2	49·6
Greenwich ...	1866·2	3	56·64	1866·2	3	49·8
Paris	1871·9	2	(56·82)	1871·8	1	49·5
Greenwich ...	1873·6	5	56·67	1873·9	7	49·8
Camb. (Eng.)...	1874·0	4	56·71	1874·0	4	49·6
Romberg ...	1875·3	5	56·74	1875·3	5	49·6
Greenwich ...	1882·1	3	56·72	1881·3	4	50·5
Washington ...	1885·2	15	56·70	1885·2	15	49·5
Dunsink ...	1886·2	4	56·67	1886·2	4	50·3
Cincinnati ...	1889·8	4	56·64	1889·8	4	50·0
Glasgow ...	1891·9	2	56·78	1891·9	2	49·8
Greenwich ...	1894·7	17	56·72	1894·4	20	49·8
Greenwich ...	1897·0	1	(57·00)	1897·0	1	50·0
Cincinnati ...	1899·8	3	56·72	1899·8	3	49·2
Greenwich ...	1905·8	4	56·77	1905·7	4	50·0

The relative masses may be derived in a different manner. Besides the two stars A and B which make up the binary, there is a third star, C, of the ninth magnitude, whose position with respect to A has been repeatedly measured. Its apparent motion

with respect to A for the last fifty years is exactly equal and opposite to the proper motion of A, and may therefore be considered as fixed in the sky. If, however, the star A has an orbital motion, the apparent motion of C, instead of being equal and rectilinear from year to year, should be affected by this change of position of A. The measures may be treated in two ways; but in each method it is first requisite to change the polar coordinates θ and ρ into differences of right ascension and declination. This being done we may correct for proper motion of A and solve the equations

$$\rho' . \cos \theta' = c\rho . \cos \theta + \Delta\delta$$
$$\rho' . \sin \theta' = c\rho . \sin \theta + \Delta\alpha$$

where ρ and ρ' are distances of B and C from A
θ „ θ' „ position-angles „ „ „
$\Delta\alpha$ „ $\Delta\delta$ „ differences „ R.A. and Decl. of C and the centre of gravity of A and B

and $c\rho$ is the distance of A from the centre of gravity.

	Position-angle.	Distance.	Diff. R.A. reduced to 1900.	Diff. N.P.D. reduced to 1900.
1851·96	114·1	33·03	9·77	33·58
52·67	113·9	32·60	9·49	33·13
65·91	92·1	18·89	9·43	32·72
68·77	82·4	17·03	9·04	32·83
69·98	77·8	16·13	9·23	32·86
70·65	74·4	15·47	9·45	32·89
74·66	54·4	13·92	9·71	32·91
76·77	40·3	14·02	10·22	33·43
77·94	39·8	14·00	9·36	32·36
78·54	33·6	14·40	9·87	33·03
78·74	32·8	14·76	9·64	33·24
79·27	30·4	14·96	9·65	33·23
80·57	25·0	15·41	9·64	33·01
81·61	20·6	16·34	9·51	33·32
82·73	16·7	17·25	9·38	33·45
83·54	11·3	17·34	[10·25]	33·16
86·24	7·6	19·84	8·74	33·17
86·99	6·1	21·15	8·78	33·78
88·67	0·9	21·71	9·05	32·77
89·61	358·6	22·69	9·28	32·86
90·52	356·7	23·59	9·45	32·83

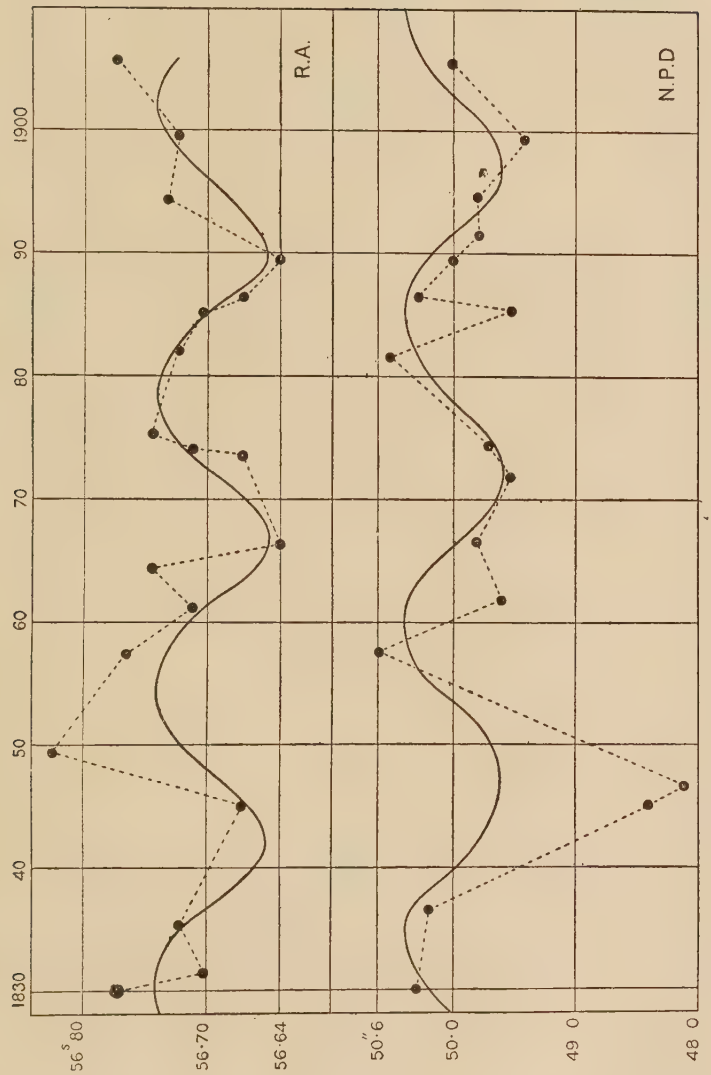

Fig. 2.—Diagram showing R.A.s and N.P.D.s of 85 *Pegasi* from T.C. Observations.

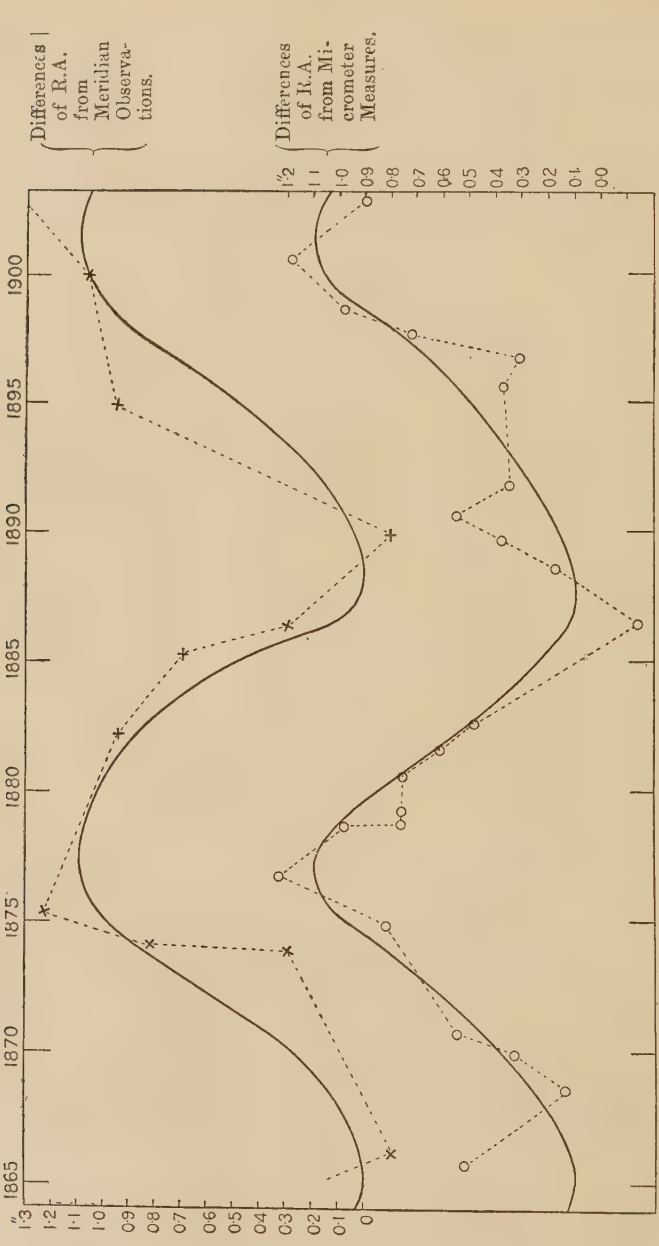

FIG. 3.—Comparison of Meridian and Micrometer Places of 85 *Pegasi*.

	Position-angle.	Distance.	Diff. R.A. reduced to 1900.	Diff. N.P.D. reduced to 1900.
	°	"	"	"
1891·84	354·4	24·90	9·25	32·78
95·53	349·0	29·17	9·27	33·01
96·75	347·8	30·48	9·20	32·98
97·69	345·9	31·62	9·62	32·93
98·57	344·4	32·68	9·86	32·92
1900·66	342·9	35·15	9·74	32·97
00·86	342·3	36·60	10·41	34·03
02·73	341·3	37·64	9·79	32·98

The graphical method has, however, the advantage of exhibiting more clearly the movement of the principal star. The corrected differences of right ascension and declination are disposed in two curves, and the amplitude of the curves compared with the similar extent of change in the apparent orbit. These two curves are not given, but the third diagram shows the R.A. curve and the corresponding portion of the first curve in fig. 2 to show the close agreement.

Thus if 1″·35 be the extent in R.A. of the relative orbit and 1″·00 be the extent in R.A. of the difference in R.A., we have

$$\frac{1{\cdot}35-1{\cdot}00}{100} = \frac{7}{20} = \frac{\text{Mass A}}{\text{Mass B}}$$

from the right ascension differences, and similarly

$$\frac{1{\cdot}05-0{\cdot}75}{{\cdot}75} = \frac{2}{5} = \frac{\text{Mass A}}{\text{Mass B}}$$

from the N.P.D., giving equal weight to each ; then

$$\frac{\text{Mass A}}{\text{Mass B}} = \frac{3}{8}$$

Incidentally it gives the period of 85 *Pegasi* as 24·0 years, a result deserving of equal confidence with that derived from direct measures of A and B.

The star C was employed by Brünnow to determine the parallax of 85 *Pegasi*, which he found to be 0″·054.

The parallax according to Flint is 0″·03. Spectrum, D.C. 1890—E (Type II.).

New Double Stars. By the Rev. T. E. Espin, M.A.

The Spring has been unusually good for observing; not only have there been a great number of fine nights, but the definition has been, on the whole, excellent. The stars are entirely situated between 30° and 40° N. declination.

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Date.	Nights.	
270	35°, 436	2 10·4	+36° 0	358·8	2·95	9·2	12·0	06·98	2	BC
				343·7	42·35	...		06·98	2	AB
271	34, 459	26·7	34 48	66·4	1·26	9·5	10·0	06·79	1	
272	35, 551	40·2	35 54	77·0	3·55	9·1	10·5	06·83	2	
273	34, 633	3 16·7	35 0	356·4	2·81	9·1	9·1	06·09	2	
274	...	22·1	35 37	142·4	3·18	9·3	9·4	06·11	3	
275	36, 737	36·3	36 49	297·9	3·41	9·3	10·2	06·13	3	
276	37, 819	37·3	37 40	282·8	7·81	8·0	13·8	06·14	3	
277	34, 741	43·8	34 31	289·7	7·40	10·0	14·0	06·09	2	BC
				142·5	30·24	A =	7·0	06·09	2	AB
278	39, 937	4 2·4	39 54	168·8	1·98	7·7	9·2	06·14	3	
279	34, 866	15·9	35 1	242·6	3·24	9·2	11·2	06·82	2	
280	39, 1201	5 4·9	39 49	302·8	3·19	9·0	10·5	06·14	1	
281	40, 1254	13·5	40 13	215·4	2·29	9·0	9·6	06·14	2	
282	...	21·7	33 44	114·6	1·99	9·1	10·3	06·09	3	
283	39, 1407	39·9	39 56	S.	2 ±	9·5	9·8	06·13	1	
284	37, 1345	46·5	37 24	184·4	4·73	9·0	11·0	06·07	1	
285	38, 1375	59·6	38 55	167·4	2·33	9·0	9·3	06·15	2	
286	39, 1550	6 6·9	39 45	64·0	2·83	9·0	9·5	06·15	2	
287	37, 1476	12·6	37 21	255·5	6·05	9·0	12·5	06·15	1	
288	39, 1600	14·9	39 11	148·9	4·40	9·0	9·3	06·16	1	AB
				273·8	13·42	C =	12·0	06·16	1	AC
289	39, 1825	55·9	39 6	98·9	1·97	9·4	9·7	06·18	2	
290	36, 1606	7 14·4	36 55	312·4	4·74	9·2	10·0	06·10	3	
291	32, 1667	56·4	32 17	343·4	7·12	8·5	10·2	06·19	2	
292	38, 3876	8 4·9	38 24	165·7	2·50	8·5	9·1	06·18	2	
293	32, 1705	10·5	32 34	214·6	4·76	9·0	9·4	06·22	2	
294	36, 1873	42·8	36 31	162·5	1·70	9·0	9·2	06·11	2	
295	35, 1874	42·8	35 21	306·8	3·58	9·1	11·5	06·74	3	
296	36, 1932	9 6·2	36 47	123·3	1·83	11·5	12·5	06·13	2	BC
				174·2	19·88	A =	8·2	06·13	2	AB
297	39, 2241	18·9	39 11	40·1	3·60	8·6	10·7	06·15	2	
298	39, 2242	19·4	+39 2	308·7	7·89	8·8	11·2	06·17	2	AB

No.	B.D.	R.A. 1900. h m	Decl. ° ′	P. °	D. ″	Mags.		Date.	Nights.	
298	39, 2242	9 19·4	+39 2	169·9	3·71	10·0	11·0	06·17	2	CD
				318·9	92·17	...		06·17	2	AC
299	35, 2017	25·8	34 56	217·1	4·78	11·2	11·5	06·26	2	BC
				360·0	55·87	A=	9·0	06·26	2	AB
300	35, 2021	27·6	35 36	142·4	2·04	9·2	10·3	06·27	2	
301	40, 2245	36·5	40 45	236·4	4·23	8·7	10·7	06·19	2	
302	37, 2077	10 20·6	36 58	348·4	2·63	9·2	10·7	06·17	2	
303	31, 2212	53·3	31 10	201·4	7·56	9·0	11·0	06·29	2	
304	39, 2399	55·0	39 1	89·1	5·65	9·2	10·5	06·19	2	
305	35, 2230	11 12·0	35 1	31·8	3·83	9·1	9·5	06·26	3	
306	39, 2458	33·0	39 18	341·0	6·76	8·0	11·2	06·25	2	
307	39, 2491	12 0·1	39 24	358·7	4·74	8·0	13·3	06·29	4	
308	32, 2343	13 16·5	32 30	292·2	7·06	9·1	9·1	06·28	3	
309	32, 2381	39·9	32 4	133·9	1·88	9·2	9·5	06·29	3	
310	32, 2382	40·2	31 57	nf	5±	9·2	11·0	06·27	1	
311	35, 2619	14 50·2	+34 52	288·5	3·76	8·8	9·3	06·28	2	

Notes.

271. This star was looked at again on February 9, but the condition of the air was then unsteady, and the star was not seen double.

278. A fine object, which so far has escaped detection.

282. This pair was found while measuring S 483. It is 187″ distant from B of S 483 at an angle of 30°·9.

289, 290. Angles discordant.

296. A difficult object found while measuring *h* 2483.

303. Found while measuring Σ 1492, rej. $1^m\ 10^s$ *f*.

307. Found while looking for the missing pair *h* 2595. The *comes* is extremely difficult, and can only be seen by oblique vision.

Elements of Five Long-period Variable Stars.

By A. Stanley-Williams.

As the under-mentioned five long-period variable stars have now been under observation for several years, it seems desirable to bring together the results and to derive from them fresh and more exact elements of variation. Excepting where otherwise mentioned the dates of the different maxima and minima have been derived from observations made at Hove. Some further particulars of most of these have been published in the *Astronomische Nachrichten* or the *Astronomical Journal*. With the exception of *RU Persei* the minima of all the stars are too faint for observation with a $6\frac{1}{2}$-inch telescope. *Z Lyræ*, however, does not descend much below the *minimum visibile* for this aperture.

Z Lyræ (*Ch.* 6816).

R.A. = $18^h\ 55^m\ 59^s$, Decl. = $+34°\ 48'·9$ (1900).

Maxima of this star have been observed as under :

E.	Date.	J.D. 241+	Computed Maximum.	O−C.	Mag.
				d	
−1	1899 Nov. 21 ±	4980 ±	4976·3	+3·7	...
0	1900 Sept. 2	5265	5267·3	−2·3	...
+1	1901 June 20*	5556	5558·3	−2·3	...
1	1901 July 1	5567	5558·3	+8·7	9·4
2	1902 Apr. 7 ±	5847 ±	5849·3	−2·3	...
4	1903 Nov. 10	6429	6431·3	−2·3	9·6
5	1904 Aug. 30	6723	6722·3	+0·7	9·3
6	1905 June 17	7014	7013·3	+0·7	9·9

and from these the following elements have been derived :

$$\text{Maximum} = \left\{\begin{matrix}\text{1900 Sept. } 4·3 \\ \text{J.D. } 2415267·3\end{matrix}\right\} + 291^d·0\ \text{E.}$$

The computed times of maximum and the residuals O−C are given in the fourth and fifth columns of the above table. It should be mentioned that there is a faint star ($12\frac{1}{2}$ mag.) about 12″ n.f. the variable, and easily mistaken for the latter when faint. The minimum brightness of *Z Lyræ* is probably about 14th magnitude.

RU Lyræ (*Ch.* 6895).

R.A. = $19^h\ 9^m\ 6^s$, Decl. = $+41°\ 8'·1$ (1900).

Observed Maxima.

E.	Date.	J.D. 241+	Computed Maximum.	O−C.	Mag.
				d	
2	1902 Aug. 21	5983	5987·2	−4·2	9·9
3	1903 Sept. 5	6363	6355·4	+7·6	9·4
4	1904 Aug. 28	6721	6723·6	−2·6	10·9
5	1905 Sept. 2	7091	7091·8	−0·8	10·2

The elements of variation are :

$$\text{Maximum} = \left\{\begin{matrix}\text{1900 May } 10·8 \\ \text{J.D. } 2415250·8\end{matrix}\right\} + 368^d·2\ \text{E.}$$

* Observed by Hartwig at Bamberg (*A. N.* 3744, col. 370).

TY Cygni (*Ch.* 7019).

R.A. = $19^h\ 29^m\ 51^s$, Decl. = $+28°\ 6'{\cdot}2$ (1900).

Observed Maxima.

E.	Date.	J.D. 241+	Computed Maximum.	O−C. d	Mag.
0	1900 Nov. 22 ±	5346 ±	5342·0	+4·2	...
1	1901 Nov. 11	5700	6995·8	+4·2	9·2
2	1902 Oct. 27	6050	6049·6	+0·4	8·85
3	1903 Oct. 7	6395	6403·4	−8·4	8·7
4	1904 Oct. 10	6764	6757·2	+6·8	9·1
5	1905 Sept. 19	7108	7111·0	−3·0	8·6

The following elements of variation have been derived from the foregoing maxima :

$$\text{Maximum} = \left\{\begin{matrix}\text{1900 Nov. 18}\\ \text{J.D. 2415342}\end{matrix}\right\} + 353^d{\cdot}8\ \text{E},$$

and these elements accord with the invisibility of the star on a photograph taken by Professor Max Wolf at Heidelberg on the nights of 1891 September 25–30 (total exposure 12 hours), as it should then have been near minimum brightness. The variable remains below 12th magnitude for nearly half the time.

TW Cygni (*Ch.* 7571*a*).

R.A. = $21^h\ 1^m\ 45^s$, Decl. = $+29°\ 0'{\cdot}3$ (1900).

Observed Maxima.

E.	Date.	J.D. 241+	Computed Maximum.	O−C. d	Mag.
−1	1899 Oct. 7 ±	4935 ±	4949·2	−14·2	...
+1	1901 Sept. 1	5629	5633·4	− 4·4	8·8
2	1902 Aug. 15	5977	5975·5	+ 1·5	9·6
3	1903 Aug. 3	6330	6317·6	+12·4	9·0
4	1904 June 28	6660	6659·7	+ 0·3	9·2
5	1905 May 26	6992	7001·8	− 9·8	9·0

The elements of variation are :

$$\text{Maximum} = \left\{\begin{matrix}\text{1900 Sept. 28·3}\\ \text{J.D. 2415291·3}\end{matrix}\right\} + 342^d{\cdot}1\ \text{E}.$$

The residuals in the fifth column indicate that the period is subject to a somewhat large irregularity, doubtless of a periodic character, though the observations do not yet extend over a sufficiently long interval of time to enable any accurate value of it to be derived.

RU Persei.

R.A. = $3^h\ 23^m\ 57^s$, Decl. = $+39°\ 18'·9$ (1900).

Observed Maxima.

E.	Date.	J.D. 241+	Computed Maximum.	O–C.	Mag.
				d	
0	1904 Nov. 28	6813	6814·0	−1·0	9·7
2	1905 Nov. 26	7176	7175·4	+0·6	9·4

Observed Minima.

			Computed Minimum.		
0	1904 Sept. 19	6743	6743·0	0·0	10·7
2	1905 Sept. 17	7106	7104·4	+1·6	10·4
3	1906 Mar. 14	7284	7285·1	−1·1	10·6

The adopted elements of variation derived from the above maxima and minima are :

$$\text{Maximum} = \left\{\begin{matrix}1904\ \text{Nov. } 29\\ \text{J.D. } 2416814\end{matrix}\right\} + 180^d·7\ \text{E},$$

the interval M—*m* being 71 days. The magnitude of the variable at maximum is 9·4–9·7 ; and at minimum it is 10·4–10·7.

Owing to the small range of variation, and to the fact that both the maximum and minimum brightness vary somewhat, I have not attempted to correct the above elements by means of the photographic observations published in the *A. N.* 3675. For the same reason the early B.D. observations published by Professor Küstner in the *A. N.* 3989 are not at present available for this purpose. Dr. E. Hartwig observed the star equal to B.D. +39°·803 (9·5 mag.) on 1905 October 20 (*A. N.* 4061). According to the observations made here it attained to equality with that star on 1905 October 19.

20 *Hove Park Villas, Hove :*
1906 *May* 1.

Observations of Jupiter in 1903 *and* 1905–6.

By W. F. Denning.

In *Monthly Notices*, lxiii. 331–4 (1903 April), I gave a summary of rotation periods of markings on *Jupiter* from observations at Bristol during the years 1898 to 1902 inclusive.

In continuation of these results I give the following table for the opposition of 1903 :—1388 transits were taken and 1188 of these were utilised in the determination of rotation periods :

1903.

Markings.	Rotation Period. h m s	No. of Spots.	Mean No. Rotations.
N.N. Temperate	9 55 41·1	14	217
N. Temperate	9 55 54·3	9	291
N. Tropical	9 55 31·9	3	336
Equatorial	9 50 27·9	28	561
S. Tropical	9 55 43·0	2	141
Great Red Spot	9 55 41·6	1	608
Great S. Tropical	9 55 18·7	1	548
S. Temperate	9 55 18·5	9	320
S.S. Temperate	9 55 6·0	1	210

In 1904 I made very few observations in consequence of bad health, but resumed work on *Jupiter* at midsummer 1905, and continued it until the planet approached too near to the Sun (in 1906 May) for further effective observation.

1905-6.

Markings.	Rotation Period. h m s	No. of Spots.	Mean No. Rotations.
N. Tropical	9 55 55·2	1	58
Equatorial	9 50 32·7	24	448
Great Red Spot	9 55 41·5	1	761
Great S. Tropical	9 55 19·5	1	507 P end
,, ,, ...	9 55 23·6	1	641 F end
S. Temperate	9 55 20·3	7	353

Notes on the Apparition of 1905–6.

N.N.N. Temperate Spots.—There were signs of considerable disturbance in the region far north, about 40°–65°, and extensive irregularities in the N.N.N. Temperate Belt and N. Polar Cap were visible. These consisted of large white spots with very dark areas adjoining. A number of transits were taken, but they were insufficient to render the identifications certain and to enable perfectly safe rotation periods to be derived.

N.N. Temperate and N. Temperate Belts.—Very faint and only just traceable as delicate pencil-like shadings.

N. Tropical Spot.—This was discovered on 1906 April 10, and probably represented an extensive disturbance which had occurred only a few days previously; for I had examined the same region on April 3 and 7 without noticing anything unusual. When first seen the spot was very dark and distinct, and formed the *f* extremity of a slanting belt running to W.S.W. from the spot and joining the N. Equatorial Belt. Contemporary with

this disturbance the N. Equatorial Belt, which in previous months had been extremely feeble, exhibited signs of rapid development, and especially in that region *p* the N. Tropical Spot and N. of the Great Red Spot. But the changes referred to, occurring, as they did, near the close of the apparition, could only be followed during a comparatively short interval (April 10 to May 4). The longitude of the N. Tropical Spot increased from 75° to 80° (System II.) in the twenty-four days included in the period alluded to, becoming increasingly conspicuous, so that it could be as definitely observed as a satellite-shadow, and was readily visible at mid-transit on April 20 at $5^h\ 22^m$, nearly two hours before sunset. Its rotation period was about 14 seconds longer than that of the Red Spot, and it evidently participated in the motion of the N. Temperate current rather than in that of the usual N. Tropical Spots, which rotate in about 10 seconds less time than the Red Spot.

Equatorial Spots.—These consisted of the normal white and dark markings alternating on the N. side of the Great S. Equatorial Belt. Their mean period of rotation proved to be a few seconds in excess of the rate exhibited during the previous eight years.

Great Red Spot.—This appeared very faint and indefinite except under very sharp definition, when its oval outlines could be distinctly traced. The motion was a little irregular, but the spot was certainly not accelerated in any striking degree, when the Great S. Tropical Spot closely approached its eastern side. The junction of the *f* end of Red Spot with *p* end of the S. Tropical Spot occurred on about March 5, for no division between them could be remarked on that date.

Great S. Tropical Spot.—A transit of the ends of this extensive disturbance, obtained on 1905 August 7, indicated its length as 43°·5, but a rapid distension thereafter occurred, for on September 13 the length was 49°·6, and on October 19, 58°·6. A further increase was noted in ensuing months, but it was of relatively trifling character. This longitudinal growth of the marking evidently occurred at the following end; for while the *p* end rotated at the normal rate, the *f* end showed a considerable retardation, the period being 4 seconds greater. The dark material of the fore part of the Spot was passing S. of the Red Spot in March and April, and the dark front of the former was not due on the *p* side of the latter until about the middle of 1906 May; but as early as April 19 there was a dusky patch extending from 351° to 11°, and well in advance of the Red Spot. I had observed this indistinctly on several previous occasions in April, but its feebleness caused me to question its relationship with the much darker aspect of the old S. Tropical Spot. If identity is assumed then its translation from the *f* to the *p* side of the Red Spot must have been exceedingly rapid and fully bears out the suggestion of Major P. B. Molesworth ('*Monthly Notices*, lxvi. p. 102).

A brilliant white spot usually precedes the Great S. Tropical disturbance, and the former apparently passed round the *northern* side of the Red Spot. It was seen on 1906 February 21 N. *f* the Red Spot, and its longitude on four other nights was : March 5, 47°·8 ; April 12, 23°·2 ; April 19, 24°·1 ; April 29, 17°·1.

S. Temperate Spots.—As usual there were several large white spots fringing the S. border of the S. Temperate Dark Belt. Some of these markings appear to have existed for a long period, though little effort has been made to identify the same individual features during many successive apparitions. This, however, now admits of easy attainment from the abundance of materials accumulated in recent years. In *The Observatory*, 1904, September (vol. xxvii. p. 345), I gave some data with regard to two of these objects which had been followed during several years. They are still visible at the present time.

During the past apparition the S. Tropical Zone usually appeared to be the brightest region of the disc, whereas in 1903 the equatorial zone was more luminous than any other.

Observations of *Jupiter* in bright sunshine are sometimes very effective. On many occasions I have seen the principal markings beautifully defined at about 2^h or $1\frac{1}{2}^h$ before sunset. But the best time for examination of Jovian detail is near the time of sunset, when the spots and irregularities in the belts are often presented with remarkable distinctness. Whenever daylight observations of *Jupiter* are attempted care must be taken to protect the tube of the telescope from the direct rays of the Sun. Equatorial mounting is not necessary, as the planet can generally be swept up in very few minutes with a low-power eyepiece.

Several observers have reported an annoying prevalence of bad definition during the past winter, but this has been quite contrary to my experience. The high altitude of the planet, as compared with its comparatively low position in the few previous years, has favourably influenced the character of the seeing, for my figures for 1903 and 1905–6 are :—

	Nights of Observation.	Very Good.	Good.	Fair.	Bad.	Very Bad.
1903	109	9	21	21	40	18
1905–6	88	17	25	22	17	7

The instrument employed in 1903 was a 10-inch With-Browning reflector, power 312. In 1905–6 I also used a $12\frac{1}{2}$-inch Calver reflector, powers 200, 315, and 440.

Bishopston, Bristol:
1906 *May* 10.

Observation of Jupiter's Sixth and Seventh Satellites from Photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1905–6.—II.

(*Communicated by the Astronomer Royal.*)

In a preliminary paper in the *Monthly Notices* for 1905 November provisional results were given from the photographs obtained up to November 7. Further photographs of the sixth and seventh satellites have since been secured, and the more accurate method of measurement indicated in the preliminary note has been applied to the whole series.

As there explained, the positions of the satellites have been measured on the photographs taken with the reflector with reference to three or four faint comparison stars (of eleventh or twelfth magnitude) symmetrically distributed round the satellite. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) in the Astronomische Gesellschaft Catalogue (Berlin Zone) from photographs (with 20 minutes' exposure) taken with the astrographic 13-inch refractor, the field sensibly free from distortion being much larger with this telescope than with the reflector, so that from twelve to sixteen reference stars were available on each plate.

As *Jupiter* moved slowly it was possible to make one reference plate serve for a number of photographs, which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the Astronomische Gesellschaft Catalogues being used for the purpose. Right ascensions and declinations of the satellites were then determined and, by comparison with the tabular positions of *Jupiter*, position angles and distances deduced.

Ten photographs to determine the errors of the tabular place of *Jupiter* were taken between 1905 November 3 and 1906 February 15 with the Astrographic Equatorial. Corrections have been deduced from these, but the discussion is not yet complete. They show, however, that the errors of the tabular place of *Jupiter* are very small, and this result is confirmed by the observations with the transit circle and with the altazimuth.

Observations of Satellite VI.

Date and G.M.T.	Apparent R.A.	Apparent Dec.	Pos. Angle.	Dist.	Exp.	No. Plate
1905. d h m	h m s	° ′ ″	° ′	′ ″	m s	
Aug. 23 13 30	4 10 42·567	+20 26 41·62	310 42·3	25 30·0	30 0	20
23 14 23	4 10 43·162	+20 26 43·01	310 33·9	25 33·2	31 30	20
Sept. 3 15 17	4 13 37·103	+20 32 25·60	291 12·1	36 54·4	40 0	20
7 14 26	4 14 20·651	+20 32 56·45	286 34·2	40 48·4	60 0	20
7 15 32	4 14 21·184	+20 32 56·78	286 32·0	40 49·8	55 0	20

Date and G.M.T. 1905.				Apparent R.A.			Apparent Dec.			Pos. Angle.		Dist.		Exp.		No. Plate.
	d	h	m	h	m	s	°	′	″	°	′	′	″	m	s	
Sept.	8	12	56	4	14	29·561	+20	32	56·02	285	34·0	41	39·8	30	0	2050
	12	14	37	4	15	1·576	+20	32	28·46	281	37·5	45	12·1	20	0	2054
	12	15	5	4	15	1·639	+20	32	29·34	281	37·5	45	14·0	20	0	2055
	12	15	42	4	15	1·783	+20	32	28·23	281	35·2	45	15·6	30	0	2056
	30	12	17	4	15	14·100	+20	21	54·75	267	49·0	54	49·3	60	0	2068
Oct.	4	12	25	4	14	47·959	+20	17	52·93	265	0·7	55	25·7	40	0	2070
	4	14	55	4	14	47·132	+20	17	45·73	264	55·8	55	26·3	34	0	2071
	4	16	38	4	14	46·515	+20	17	40·98	264	52·7	55	27·4	39	0	2072
	5	11	52	4	14	39·912	+20	16	48·24	264	18·7	55	30·6	30	0	2074
	5	14	7	4	14	39·143	+20	16	42·09	264	14·8	55	30·5	59	14	2075
	21	10	55	4	11	5·799	+19	55	34·19	251	59·5	51	52·5	45	0	2079
	21	11	54	4	11	5·047	+19	55	30·30	251	57·4	51	51·8	45	0	2080
	22	10	46	4	10	47·518	+19	54	2·67	251	6·4	51	23·2	40	0	2081
	22	12	4	4	10	46·602	+19	53	57·18	251	2·4	51	20·2	75	0	2082
	25	10	47	4	9	49·341	+19	49	17·99	248	14·7	49	43·1	30	0	2086
	25	11	47 (*a*)	4	9	48·488	+19	49	15·10	248	13·3	49	41·2	60	0	2087
	27	10	17	4	9	8·437	+19	46	5·39	246	14·0	48	30·5	40	0	2089
	27	10	58	4	9	7·783	+19	46	3·27	246	12·2	48	29·5	28	51	
	29	9	47	4	8	25·544	+19	42	49·58	244	7·0	47	13·8	30	0	2091
	29	10	20	4	8	25·051	+19	42	47·72	244	5·4	47	11·8	25	0	2092
	29	12	19*	4	8	23·396	+19	42	40·21	(243	59·9)	(47	6·6)	177	12	2093
	29	14	17	4	8	21·417	+19	42	31·64	243	55·4	47	5·8	17	0	2094
	31	10	24	4	7	39·848	+19	39	25·40	241	48·4	45	51·4	25	3	2096
	31	10	49	4	7	39·427	+19	39	23·84	241	47·9	45	51·2	20	0	
	31	12	3	4	7	38·347	+19	39	19·98	241	44·6	45	46·9	84	5	2097
Nov.	3	9	52	4	6	29·708	+19	34	22·57	238	5·4	43	42·9	30	0	2098
	3	11	48	4	6	27·898	+19	34	12·33	237	57·8	43	37·7	54	17	2100
	6	10	19	4	5	15·319	+19	29	7·67	233	53·3	41	27·0	15	0	2104
	6	10	48	4	5	14·776	+19	29	5·29	233	51·7	41	26·7	15	0	2105
	6	11	50	4	5	13·828	+19	29	0·56	233	46·0	41	23·4	70	0	2106
	7	14	12	4	4	45·963	+19	27	5·83	232	7·0	40	33·8	15	0	2110
	7	15	14	4	4	44·840	+19	27	1·23	232	3·1	40	32·2	90	0	2111
	21	9	9	3	58	35·044	+19	3	4·29	203	32·5	31	34·7	15	0	2118
	21	9	27	3	58	34·684	+19	3	3·42	203	31·1	31	34·0	15	0	
	23	12	26	3	57	35·788	+18	59	28·13	197	46·7	30	45·6	90	0	2121

* Owing to the long exposure (nearly 3 hours) the star trails are so long that the results of the measures are liable to considerable uncertainty.

(*a*) Very poor image.

Date and G.M.T.	Apparent R.A.	Apparent Dec.	Pos. Angle.	Dist.	Exp.	No. Plate.
1905. d h m	h m s	° ′ ″	° ′	′ ″	m s	
Nov. 23 13 33	3 57 34·454	+18 59 23·84	197 39·8	30 44·6	15 0	2122
23 13 53	3 57 34·064	+18 59 22·12	197 37·6	30 44·3	20 0	
24 10 31	3 57 10·277	+18 57 55·76	195 11·9	30 29·2	60 0	2126
24 11 19	3 57 9·358	+18 57 52·72	195 6·2	30 28·3	10 0	2127
24 11 33	3 57 9·069	+18 57 51·70	195 4·3	30 28·1	15 0	
27 6 52 (*b*)	3 55 51·612	+18 53 16·58	187 1·1	29 55·8	5 0	2129
27 7 1 (*b*)	3 55 51·502	+18 53 16·14	186 58·4	29 55·5	10 0	
29 9 38	3 54 53·828	+18 49 54·97	180 47·2	29 50·0	20 0	2131
29 9 55	3 54 53·525	+18 49 53·57	180 44·6	29 50·2	10 0	
29 10 33	3 54 52·844	+18 49 51·23	180 39·1	29 50·1	30 0	2132
29 11 17	3 54 52·062	+18 49 48·63	180 32·4	29 49·9	19 0	2133
Dec. 19 6 56	3 46 49·665	+18 24 32·09	134 28·0	39 34·1	60 0	2141
19 7 50	3 46 48·978	+18 24 30·27	134 23·2	39 36·4	30 0	2142
19 8 21 (*a*)	3 46 48·517	+18 24 29·85	134 20·6	39 36·7	7 30	2143
19 8 34	3 46 48·372	+18 24 28·88	134 19·9	39 37·7	15 0	
25 7 44	3 44 56·674	+18 19 47·59	126 15·5	44 6·9	18 30	2147
25 8 41	3 44 56·088	+18 19 45·69	126 12·1	44 9·7	30 0	2149
25 9 34	3 44 55·469	+18 19 44·42	126 9·3	44 11·4	60 0	2150
25 10 21	3 44 54·884	+18 19 43·86	126 6·5	44 12·1	5 0	2151
25 10 32	3 44 54·755	+18 19 43·14	126 6·5	44 12·6	15 0	
30 7 42	3 43 39·272	+18 17 8·12	120 52·5	47 43·2	60 0	2153
30 8 42	3 43 38·726	+18 17 6·89	120 50·0	47 45·3	25 45	2154
1906. Jan. 13 9 35	3 41 30·854	+18 16 26·58	109 54·9	55 46·8	30 0	2169
13 10 11	3 41 30·715	+18 16 27·24	109 53·9	55 46·9	10 0	2170
13 10 28	3 41 30·705	+18 16 27·45	109 53·4	55 47·7	10 0	2171
15 12 11	3 41 23·599	+18 17 12·43	108 38·3	56 38·1	14 39	2174
19 10 11 (*a*)	3 41 18·661	+18 19 14·80	106 24·4	57 57·6	90 0	2175
19 11 15	3 41 18·739	+18 19 16·83	106 22·3	57 59·1	9 0	2176
19 11 25 (*a*)	3 41 18·751	+18 19 17·35	106 21·9	57 59·3	8 0	
22 9 4	3 41 22·465	+18 21 17·91	104 50·0	58 45·0	90 0	2180
22 10 8	3 41 22·597	+18 21 21·21	104 47·3	58 45·6	10 0	2181
22 10 19	3 41 22·604	+18 21 21·16	104 47·5	58 45·6	10 0	
23 8 45	3 41 25·117	+18 22 5·99	104 18·6	58 57·3	60 0	2183
23 9 47	3 41 25·307	+18 22 7·95	104 17·2	58 58·7	35 46	2184
24 8 12 (*c*)	3 41 28·508	+18 22 55·00	103 49·2	59 9·3	60 0	2185
26 10 12	3 41 37·892	+18 24 49·66	102 47·7	59 28·3	40 0	2186
30 7 46	3 42 4·286	+18 28 58·90	100 55·1	59 52·1	30 0	2189

(*a*) Very poor image. (*b*) Very faint and diffused. (*c*) Faint.

Date and G.M.T.				Apparent R.A.			Apparent Dec.			Pos. Angle.		Dist.		Exp.		No. Plate.
1906.	d	h	m	h	m	s	°	′	″	°	′	′	″	m	s	
Jan.	30	8	30 (*d*)	3	42	4·388	+18	28	59·73	100	55·9	59	50·3	42	49	2190
	30	9	31	3	42	4·767	+18	29	3·69	100	53·6	59	50·9	10	44	2191
	30	9	45 (*e*)	3	42	4·847	+18	29	3·16	100	54·5	59	51·7	7	48	2191
Feb.	12	6	49 (*b*)	3	44	44·628	+18	47	25·81	95	23·0	57	51·6	35	0	2206
	12	7	30 (*c*)	3	44	45·067	+18	47	28·80	95	22·2	57	50·5	35	0	2207
	12	8	6	3	44	45·447	+18	47	31·53	95	21·4	57	49·5	15	0	2208
	12	8	18 (*f*)	3	44	45·581	+18	47	31·75	95	21·8	57	49·3	7	0	2208
	14	7	49	3	45	20·593	+18	50	59·30	94	28·0	57	18·9	25	40	2213
	15	8	24	3	45	39·672	+18	52	48·90	94	1·2	57	1·1	35	0	2217

(*b*) Very faint and diffused. (*c*) Faint. (*d*) Very diffused.
(*e*) Very faint. (*f*) Diffused.

Observations of Satellite VII.

Date and G.M.T.				Apparent R.A.			Apparent Dec.			Pos. Angle.		Dist.		Exp.		No. Plate.
1905.	d	h	m	h	m	s	°	′	″	°	′	′	″	m	s	
Oct.	22	12	4 (*a*)	4	11	21·917	+20	22	37·71	286	35·5	41	52·3	75	0	2082
	29	12	19 (*b*)	4	9	14·364	20	12	25·29	286	39·5	31	34·7	177	12	2093
	29	14	17	4	9	12·664	20	12	16·70	286	37·5	31	27·8	17	0	2094
	31	12	3	4	8	33·873	20	9	13·19	286	43·8	28	22·1	84	5	2097
Nov.	3	11	48 (*c*)	4	7	29·995	20	4	11·40	286	57·6	23	18·0	54	17	2100
	6	11	50 (*c*)	4	6	22·872	19	58	53·38	287	29·0	17	53·1	70	0	2106
	7	15	14	4	5	55·910	19	56	49·95	287	42·7	15	55·2	90	0	2111
	23	12	26 (*d*)	3	59	14·996	19	26	2·13	100	58·6	14	17·0	90	0	2121
	24	10	31	3	58	50·433	19	24	13·13	101	17·1	15	57·3	60	0	2126
	29	10	33	3	56	37·172	19	14	21·63	102	18·1	24	51·4	30	0	2132
Dec.	19	6	56 (*c*)	3	48	11·497	18	40	40·88	103	35·3	48	56·8	60	0	2141
	25	8	41	3	45	56·035	18	33	19·26	104	3·1	51	20·2	30	0	2149
	25	9	34	3	45	55·293	18	33	18·07	104	2·0	51	20·9	60	0	2150
	30	7	42 (*e*)	3	44	16·984	18	28	44·11	104	27·1	51	29·2	60	0	2153
	30	8	42	3	44	16·128	18	28	42·34	104	27·4	51	27·7	25	45	2154
1906.																
Jan.	19	10	11 (*e*)	3	40	0·128	18	24	18·82	107	8·0	38	39·6	90	0	2175
	23	8	45 (*f*)	3	39	42·537	18	26	9·57	107	57·1	34	27·1	60	0	2183
	23	9	47 (*g*)	3	39	42·536	18	26	10·59	107	57·4	34	26·1	36	46	2184
	26	10	12	3	39	37·292	18	28	9·57	108	43·7	31	1·3	40	0	2186

(*a*) Very diffused.
(*b*) Very diffused. Star trails 66″ long, and consequently subject to uncertainty in measurement of their positions.
(*c*) Very faint. (*d*) Faint and diffused. (*e*) Very poor photograph.
(*f*) Extremely faint. (*g*) Faint.

Royal Observatory, Greenwich:
1906 *May* 11.

Observations of Uranus at Windsor, New South Wales. By John Tebbutt.

The following are the results of comparisons of *Uranus* made with *I Sagittarii* by means of the filar micrometer on the 8-inch equatorial. The estimated centre of the planet's disc was chosen for observation. The last two columns contain a comparison of the results with the transit ephemeris on page 280 of the *Nautical Almanac*.

1905.	Windsor Mean Time.	Planet−Star		No. of Comps.	Reduction to Apparent Place.		Parallax Corrections.		Concluded Geocentric Apparent Place of Planet.		Obs.−Cal.	
		$\Delta\alpha$.	$\Delta\delta$.		α.	δ.	α.	δ.	α.	δ.	α.	δ.
	h m s	s	″		s	″	s	″	h m s	° ′ ″	s	″
July 15	8 47 29	+14·34	+47·2	10	+2·81	+8·9	−0·01	−0·1	18 6 12·61	−23 42 20·1	+0·11	−2·0
,, 16	8 45 45	+ 4·56	+45·2	6	+2·81	+8·9	−0·01	−0·1	18 6 2·83	−23 42 22·1	−0·02	−2·3
,, 17	8 13 37	− 4·72	+44·4	20	+2·81	+8·9	−0·02	−0·1	18 5 53·54	−23 42 22·9	+0·05	−1·5
,, 21	7 41 38	−41·78	+38·3	20	+2·81	+8·8	−0·02	−0·1	18 5 16·48	−23 42 29·1	+0·07	−2·0
,, 22	7 40 34	−50·86	+37·3	20	+2·81	+8·8	−0·02	−0·1	18 5 7·40	−23 42 30·1	+0·04	−1·7
,, 23	7 40 57	−59·90	+36·2	20	+2·81	+8·8	−0·02	−0·1	18 4 58·36	−23 42 31·2	−0·05	−1·5

The mean place of the comparison star for 1905·0 is $\alpha = 18^h\ 5^m\ 55^s{\cdot}47$, $\delta = 23^\circ\ 43'\ 16''{\cdot}1$. It is derived from the following authorities: Argentine Gen. Cat. 1875, No. 24755; Greenwich Ten-year Cat. 1880, No. 2884; Stone, 1880, No. 9907; Greenwich 2nd Ten-year Cat. 1890, No. 4509; and Radcliffe Cat. 1890, No. 4748.

Observatory, Peninsula, Windsor, N.S. Wales:
1906 *March* 8.

Observations of Comet c 1905 made at the Natal Observatory, Durban.

(*Communicated by E. Nevill.*)

The following observations were made by Mr. Rendell by means of a cross-bar micrometer with the equatorial refractor, aperture 8 inches, focal length 10 feet, magnifying power 50.

Date. 1906.	Greenwich Mean Time.	Apparent Difference Comet—Star. R.A.	Apparent Difference Comet—Star. N.P.D.	Comet's Approx. Hour-angle.	No. of Comparisons.	Comparison Star.
	h m s	m s	′ ″	h m		
Feb. 16	6 7 7	−0 46·83	−3 22·8	5 20 W	6	*a*
18	6 4 47	+1 22·02	+0 56·2	5 13 W	4	*b*
20	6 16 6	+1 33·31	−3 21·1	5 19 W	6	*c*
21	5 57 5	−1 1·14	−3 3·1	4 58 W	6	*d*
22	5 51 24	−0 11·24	−1 2·7	4 50 W	6	*e*
25	5 45 8	+0 24·64	−4 2·9	4 39 W	4	*f*

Comparison Stars.

a	B.D.−15° No. 119	R.A. 0 h 32 m 34·8 s	N.P.D. 105° 31′·8 (1855·0)
b	B.D.−13° No. 142	0 44 0·9	103 54·3 „
c	B.D.−12° No. 196	0 56 33·3	102 26·8 „
d	B.D.−11° No. 224	1 5 1·9	101 42·8 „
e	B.D.−10° No. 270	1 9 59·9	100 57·3 „
f	B.D.− 8° No. 265	1 25 37·0	98 54·0 „

[Star *b* = Lalande 1422 = Paris 1074; star *d* = Lalande 2164 = Paris 1542.]

Notes.

The observations were taken each evening as soon as possible after sunset.

The comet, although rather low in the sky, and somewhat faint, was visible on each occasion in the 3-inch finder. No stellar nucleus was seen.

The nebulosity, in diameter about 1′, was fairly dense.

Faint extensions could not be detected.

On February 20 the comet appeared to be growing fainter and a little more diffused.

Estimated to be almost as faint as B.D.−12° No. 202 (mag. 10).

On February 21 it was estimated to be a little brighter than B.D.−11° No. 221 (mag. 9·7).

The observations have not been corrected for refraction or parallax.

Natal Observatory, Durban:
1906 *March* 23.

Note on the Parallax and Proper Motion of the Central Star in the Annular Nebula in Lyra. By Burt L. Newkirk.

(Communicated by Professor A. O. Leuschner.)

In a recent article on the annular nebula in *Lyra* (*Monthly Notices*, vol. xlvi., page 106) Professor Barnard draws the following conclusion concerning my investigation of the parallax of the central star: "As Dr. Newkirk's parallax for the central star depends upon the proper motion which he determined, and which is shown not to exist, the parallax itself must be fallacious."

This conclusion is not justified unless a solution of the equations of condition with the proper motion terms omitted indicates that no measurable parallax exists. I have made such a solution with the following results from the eight pairs of comparison stars:—

Pair	π
	"
1– 2	0·00
3– 4	+0·17
5– 6	+0·07
7– 8	+0·02
9–10	+0·13
11–12	+0·06
13–14	+0·05
15–16	+0·07

Weighted mean parallax = $+0''\cdot067 \pm 0''\cdot02$ mean error.

The result obtained when proper motion terms are included in the equations of condition is

$$\pi = +0''\cdot10 \pm 0''\cdot02 \text{ M.E.}$$

If instead of averaging these eight values equations of condition be set up for the simultaneous determination of the parallax and the effect of chromatic dispersion, the above value of the parallax is reduced by $0''\cdot003$ only.

I am fully aware of the uncertainty which attaches itself to investigations of stellar parallax, and realise the desirability of a thorough test of my results. An investigation of proper motion alone, however, does not seem likely to throw much light on the value of the parallax.

Most of the plates used in my parallax investigation were exposed during the years 1899 and 1900, and the series could not therefore form a good basis for an investigation of proper motion. This is explicitly stated on page 15.

The proper motion obtained from the photographic measures was :

$$\Delta\alpha = -0^s{\cdot}01 \pm 0^s{\cdot}0036$$
$$\Delta\delta = -0''{\cdot}00 \pm 0''{\cdot}045$$

A comparison of measures of the distance nucleus to star a, made at various times by Burnham, Scheiner, Barnard, and Leavenworth gives the values :

$$\Delta\alpha = +0^s{\cdot}01 \pm 0^s{\cdot}0013\ *$$
$$\Delta\delta = +0''{\cdot}18 \pm 0''{\cdot}037$$

The weighted mean is, in the case of $\Delta\alpha$, practically independent of my photographic measures.

The evidence of proper motion which was presented as a supplement to the parallax investigation rested, therefore, mainly on observational material not used in the parallax determination.

Students' Observatory, University of California, U.S.A.:
1906 *March* 6.

Some Considerations regarding the Number of the Stars. By Winifred Gibson, B.Sc., Jessel Student, University College, London.

(*Communicated by Professor* ***Karl Pearson****, F.R.S.*)

(1) The aim of the following investigation is to consider whether any suggestion of a limitation of the number of the stars can be inferred from the very full statistics we already possess with regard to stars of the lower magnitudes.

If the universe be at all comparable with the atomic congregations such as we find the physicist postulating in the kinetic theory of gases, we might not unreasonably assume that some one or more characteristics of the stars may obey a chance law of distribution in their frequency ; and hence, if we knew fairly well the frequency distribution for stars of the lower magnitudes, we might hope to construct somewhat roughly the total frequency distribution, and thus possibly obtain a measure, if only the crudest, of the total number of stars.

Unfortunately, however, there is only one character of the stars of which we have as yet anything like ample enough statistics for an inquiry of this kind, namely, the frequency of stars up to about the tenth magnitude ; and even on this point star catalogues and authors are far from being in complete agreement. Proper motions, parallaxes, colours, spectroscopic measurements, &c. have been ascertained at present for too

* By an unfortunate error this was given the negative sign, thus seeming to agree with the results of the photographic measures.

few stars to render it at all hopeful to consider frequency distributions for such characters, and even such determinations of these properties as have already been made are far too doubtful and unreliable to form the basis of accurate statistical reasoning.

We are therefore thrown back on magnitude as the only character for which at present a frequency distribution is practicable. But it is impossible to consider magnitude as a quantity which in nature itself could be the basis of a chance distribution of frequency. It is largely physiological, and is influenced by a variety of causes which it is difficult to disentangle, and of which one or more might be the basis of a real chance distribution. Thus the frequency of the stellar distances from the Sun or from any other point in the universe might conceivably follow a chance distribution ; the size of the stars might do so ; their colour or the intensity of their lights ; but it is hardly conceivable that a scale of pure magnitude could have any meaning in physical nature. If, however, we could obtain a scale in which successive magnitudes should correspond to equal increases in the amount of light, we might possibly obtain a chance distribution of frequency.

Starting from Fechner's Law of Sensation, that the increment of sensation is proportional to the relative increase of the excitation, we should have

$$\delta S \propto \delta l/l$$

where S is the sensation of light, which is really what we are measuring with magnitude, and l is a physical measure of the intensity of light.

It follows from this that if m_1 and m_2 be two magnitudes corresponding to physical amounts l_1 and l_2 of light,

$$m_2 - m_1 = C(\log l_1 - \log l_2)$$

where C is a constant, or

$$l \propto C'/a^m \quad \ldots \quad \ldots \quad \ldots \quad \text{(i.)}$$

The validity of this reasoning as well as Fechner's Law itself may well be called in question, and it is enough merely to define the relation of magnitude to amount of light by (i.), where astronomers are agreed to give to a Pogson's value 2·51189, so that $\log_{10} a = \cdot 4$. Now the magnitudes as determined by the amounts of light received by (i.) may give a chance distribution when their frequencies are simply plotted to this light, or it may be needful to replace light by some other character of which it is a function, such as the inverse square of the distance d from the Earth. In the latter case we have

$$m_1 - m_2 = \frac{2}{\log a}(\log d_1 - \log d_2)$$

$$= 5(\log d_1 - \log d_2) \quad \ldots \quad \ldots \quad \text{(ii.)}$$

According to this there would be an invariable relation between magnitude and distance which should manifest itself in a very high correlation between the two characters magnitude and parallax.

Again, if stars were scattered at random throughout space, we should have the number n inside a sphere of radius d proportional to d^3, or

$$m_1 - m_2 = \tfrac{5}{3}(\log n_1 - \log n_2) \quad \ldots \quad \ldots \quad \text{(iii.)}$$

But the accuracy of (iii.) would, unless a corrective combination of errors were called into play, be no greater than that involved in the hypothesis that the relative light of the stars is due wholly to their distances.

(2) So far we have been dealing purely with somewhat vague hypotheses—Fechner's Law, the variation of stellar light with the square of the distance, which neglects the absolute intensity of light of individual stars, and the random scattering of stars in space. A necessary fundamental step seemed to be the testing of such matters by the modern methods of statistics. Unfortunately the amount of material at our disposal is comparatively small. Newcomb, in his book on the Stars,* gives the parallax of 72 stars, and this is the longest list I have been able to find. A considerable number of these stars are noted as being of rather doubtful parallax ; still this material is all we have at present to justify what appear otherwise to be rather sweeping conclusions.

The first step taken was the investigation of the correlation of parallax and magnitude of stars. As noted before, if the magnitude of stars is closely related to their distance, we should expect this correlation to be a high one.

Table I. is the resulting correlation table. In this table the column 0″·30, say, contains all stars with a parallax equal to or greater than 0″·30 and less than 0″·35 ; that is, since the stars' parallaxes are tabled to two decimal places only—all stars between 0″·295 and 0″·345—the row 3, say, contains all stars of magnitude 3 to 3·5. The statistical constants of this table were obtained in the usual manner *on the assumption that magnitude is a uniformly graded variable.* There resulted :

Mean parallax = 0″·145±0″·011

Standard deviation in parallax = 0″·134±0″·007

Mean magnitude = 4·026±·218

Standard deviation in magnitude = 2·741±·154

Correlation between parallax and magnitude = ·103±·079

This somewhat astonishing result indicates that a star of low magnitude, *i.e.* a bright star, is on the average associated with a low parallax, *i.e.* a great distance. The probable error of the result indicates, however, that for the stars dealt with *there is no*

* *The Stars: a Study of the Universe.* Murray, 1902.

sensible relation between magnitude and parallax. In other words, assuming we may treat magnitude as a uniformly graded variable, we are compelled to conclude that on the results available, which do not go beyond stars of the ninth magnitude, there is no evidence whatever that magnitude is in any way associated with distance. The bearing of this result on differential methods of ascertaining parallax, if brightness and not proper motion be taken as a test of relative distance, is considerable.

TABLE I.

Correlation of Parallax and Stellar Magnitude.

Value of Magnitude.	Value of Parallax.																Totals.
	″·00.	″·05.	″·10.	″·15.	″·20.	″·25.	″·30.	″·35.	″·40.	″·45.	″·50.	″·55.	″·60.	″·65.	″·70.	″·75	
−1·5	...	...	...	...	...	...	...	1	...	...	...	...	...	...	...	...	1
−1	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1
− ·5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	0
0	2	1	1	...	...	...	...	...	...	...	...	...	...	...	...	1	5
·5	2	1	...	...	1	...	1	...	...	...	...	...	...	...	...	...	5
1	3	1	2	...	...	...	...	...	...	...	...	...	...	...	...	...	6
1·5	1	...	...	...	1	...	...	...	...	...	...	...	...	...	...	...	2
2	2	2	...	1	...	...	...	...	...	...	...	...	...	...	...	...	5
2·5	...	1	1	...	...	...	...	...	...	...	...	...	...	...	...	...	2
3	...	2	2	...	...	...	...	...	...	...	...	...	...	...	...	...	4
3·5	...	...	...	...	1	...	1	...	1	...	...	...	...	...	...	...	3
4	1	...	2	1	1	...	...	...	...	...	...	...	...	...	...	...	5
4·5	...	1	...	1	1	1	1	1	...	...	...	...	...	...	...	...	6
5	1	1	2	...	...	...	...	...	...	...	...	...	...	...	...	...	4
5·5	1	2	...	1	...	...	...	...	...	...	...	...	...	...	...	...	4
6	2	...	1	...	...	...	...	...	...	...	...	...	...	...	...	...	3
6·5	...	...	1	1	...	...	...	...	...	...	...	...	...	...	...	...	2
7	...	1	...	1	...	1	...	...	...	...	...	...	...	...	...	...	3
7·5	1	1	...	...	1	...	...	...	...	1	...	...	...	...	...	...	4
8	1	1	...	1	...	...	1	...	...	...	...	...	...	...	...	...	4
8·5	...	...	...	...	1	...	...	1	...	...	...	...	...	...	...	...	2
9	...	...	...	...	...	1	...	...	...	...	...	...	...	...	...	...	1
Totals	18	15	12	7	7	3	4	3	1	1	...	...	...	...	...	1	72

The question then naturally arose as to whether the fault lies with parallax or magnitude. To test this the correlations between parallax and (*a*) proper motion in R.A., and (*b*) proper motion in declination, were obtained for the same 72 stars.

I have most heartily to thank Dr. A. Lee for working out

these values : they were obtained *without grouping* from the second moments and product moments of the whole material. The results were as follows :—

Mean parallax $= 0''\cdot141 \pm 0''\cdot011$*

Standard deviation of parallax $= 0''\cdot132 \pm 0''\cdot007$

Mean proper motion R.A. $= 0^s\cdot106 \pm ^s\cdot012$

Standard deviation of proper motion R.A. $= 0^s\cdot154 \pm ^s\cdot009$

Mean proper motion declination $= 0''\cdot657 \pm ''\cdot070$

Standard deviation of proper motion in declination $= 0''\cdot878 \pm ''\cdot049$

Correlations :—

Parallax and proper motion in R.A. $= \cdot435 \pm \cdot064$

Parallax and proper motion in declination $= \cdot414 \pm \cdot066$

Proper motion in R.A. with proper motion in declination $= \cdot293 \pm \cdot073$

Now these results are distinctly lower than we might have anticipated. We might expect *à priori* a very high association between parallax and proper motion ; the results show it to be quite significant and important, but not halfway up the scale of correlation. This, perhaps, may confirm the view that many parallaxes are unreliable ; but it may also serve to show that some extremely distant stars have very large proper motions.

Before we leave this point it is worth putting on record the most probable value of the parallax of a star whose proper motions in right ascension and declination have been measured. It was found in the usual manner and is given by the equation below. Π = probable parallax in seconds of angle ; p_{AR} = proper motion in R.A. in seconds of time, p_δ = proper motion in declination in seconds of arc. The formula is then

$$\Pi = ''\cdot079 + ''\cdot294 p_{AR} + ''\cdot047 p_\delta \pm ''\cdot076 \quad \ldots \quad \text{(iv.)}$$

In using this formula no attention, of course, must be paid to the *sign* of the proper motions.

Now any determination of parallax will only differ from the mean value $0''\cdot141$ with a probable error of $0''\cdot089$. Thus by predicting parallax from proper motion we only reduce the probable error of the estimate $0''\cdot089$ to $0''\cdot076$, no very great improvement. We therefore conclude that the knowledge of the proper motion of a star in right ascension and declination does not enable us to form any good estimate of what its parallax is likely to be.

* Allowing for the effect of grouping, these results for parallax are in agreement with those given above.

The following table illustrates the point :—

Table of Observed and Estimated Parallaxes.

Star.	Observed Parallax.	Estimated Parallax.	Star.	Observed Parallax.	Estimated Parallax.
	″	″		″	″
α Centauri	0·75	0·256	Formalhaut	0·13	0·141
Sirius	·37	·142	Aldebaran	·11	·090
γ' Draconis	·32	·092	Capella	·09	·102
Procyon	·30	·147	Pollux	·06	·096
β Cassiopeiæ	·15	·108			

It was not, of course, possible to let the question of magnitude and distance rest at the point indicated above. Magnitude is a physiologically and not a physically graded variate, *i.e.* a unit of magnitude high in the scale is not physically equal to one low in the scale. On this account it seemed advisable to reinvestigate the whole matter, in this case taking light units as given by equation (i.), with Pogson's value for a, and correlating once more with parallax.

The light of a star of the tenth magnitude was taken as the unit of light, or we have

$$l = \frac{10000}{(2{\cdot}51189)^m} \quad \ldots \quad \ldots \quad \ldots \quad \text{(v.)}$$

The vertical coordinate was now taken to be l instead of m, and the correlation again worked out. From this we have :

Mean parallax $= 0''{\cdot}145 \pm ''{\cdot}011$

Standard deviation in parallax $= 0''{\cdot}134 \pm ''{\cdot}008$

Mean amount of light $= 2669{\cdot}21 \pm 470{\cdot}66$ units

Standard deviation in light $= 5921 \pm 332{\cdot}82$ units

Correlation between light and parallax $= {\cdot}094 \pm {\cdot}079$

This gives a positive correlation—which is insensible having regard to its probable error—between intensity of light and magnitude of parallax. The star α *Centauri*, which has the exceptionally large parallax of $0''{\cdot}75$, was obviously very influential on this result. On this account, although this is a well-determined parallax, it seemed worth while recalculating the correlation, omitting α *Centauri* from the table. The correlation was then found to be ${\cdot}029 \pm {\cdot}079$, which is absolutely insensible. Thus $\frac{2}{3}$ of the slight correlation already noticed is only due to the excessive parallax of this single very bright star.

We have thus found that, while the paradox of "less light greater nearness" disappears when we change our physiological into physical units of luminosity, we are still left with the same general conclusion that the parallaxes already determined give no sensible relation between intensity of light and stellar distance.

It appeared in the next place worth while considering whether the result obtained may be due to absence of linear regression. In non-technical language can it possibly be that there is a fairly close relation between light and distance, but that distance does not on the average decrease proportionately to the increase of light received? There might exist, for example, a curved regression line, giving a low coefficient of correlation, a large class of bright stars being at a great distance, or a similar class of faint stars being in our more immediate neighbourhood.

To test this we must proceed either by the method of contingency * or the use of the correlation ratio.†

Accordingly the observations were clubbed together in Table II. Alongside the number of stars falling into each group is given the number which might be expected to appear, if the relation between parallax and magnitude were purely random in its character, *i.e.* on the theory of independent probability.

From these results the coefficient of mean square contingency was calculated in the usual manner. This coefficient must lie between 0 and 1, and if the distribution were Gaussian, involving linear regression, it should be closely equal to the coefficient of correlation.

Its value was found to be ·408. This is a most sensible rise on the value ·094 previously determined for the correlation.

TABLE II.

Contingency Table between Parallax and Stellar Magnitude.

Magnitude.	Parallax.	″·00 to ″·05.	″·05 to ″·15.	″·15 to ″·25.	″·25 to ″·30.	″·30 and over.	Totals.
	−1·5 to 1	8·5 (4·75)	3 (3·96)	3 (5·01)	1·5 (2·64)	3 (2·64)	19
	1 „ 3	2·5 (3·375)	5 (2·81)	4 (3·56)	1 (1·875)	1 (1·875)	13·5
	3 „ 5	2·5 (4·625)	3 (3·85)	6·5 (4·88)	3·5 (2·57)	3 (2·57)	18·5
	5 „ 7	3 (3·00)	2·5(2·50)	4·5 (3·17)	1·5 (1·67)	·5 (1·67)	12
	7 „ 9	1·5 (2·25)	1·5(1·875)	1 (2·375)	2·5 (1·25)	2·5 (1·25)	9
	Totals ...	18	15	19	10	10	72

In forming this contingency table the dimensions were selected to give fairly uniform groups—the actual units of classification having, of course, no influence on the result, which is simply a measure of the deviations of the numbers to be expected in each group on the theory of absolute independence of the variates from the actually observed numbers.

This great increase in the association between luminosity and parallax seemed to demand some explanation, and accordingly the correlation-ratio usually represented by η was calculated.

* Pearson, "On the Theory of Contingency and its Relation to Association and Normal Correlation," *Drapers' Company Research Memoirs*, i. 1904.

† Pearson, "On the General Theory of Skew Correlation and Non-linear Regression," *Drapers' Company Research Memoirs*, ii. 1905. Dulau & Co.

If η be equal to the correlation coefficient r, the regression is linear; if it be sensibly different from r, then the correlation is non-Gaussian and sensibly skew. The means of the arrays of parallax corresponding to the five groups of luminosity in Table II. were then determined, and from these means the correlation ratio η was found in the usual way: its value was ·486. It thus amply confirms the result obtained from the coefficient of contingency. The values of both are five to six times their probable errors, and the difference between η^2 and r^2 is three to four times its probable error. We must conclude therefore that there exists :—

(i.) A significant association between distance and luminosity.

(ii.) This association is not, however, one in which, on the average, increase of distance is proportional to decrease of luminosity.

The question naturally now arises as to whether the relationship is one which could be expressed by any smooth mathematical function. Unfortunately seventy-two stars are far too few to determine a relationship of such a nature. It is, however, worth while looking at a graph of the mean parallax corresponding to each magnitude. Examining fig. 1 we see that on the present data nothing better than a horizontal straight line* at the mean parallax, or a zero correlation coefficient is likely to be found to fit the observations.

But the values of the contingency and correlation ratio indicate that the deviations from this straight line, however hopelessly irregular they appear, are not within the limits of random-sampling. The material is far too sparse to draw dogmatic conclusions from, but it pointedly suggests: (*a*) that while there appears to be no continuous relation between luminosity and distance, certainly no average proportionality between changes in the two, yet (*b*) there is some significant discontinuous relation between the two variates which removes them from the field of independent characters. For example, stars of a definite range of magnitude may be at a certain average distance, but there may be no uniform arrangement of clusters of stars of given magnitude in order of distance.†

It will be clear, therefore, that distance as rigidly related to magnitude by any such relation as (ii.) (p. 446) is not a stellar character upon which we can start with any idea of a random chance distribution. So far as any evidence is at present forthcoming, we cannot replace the magnitude variate by a corre-

* It is possible that a curve of a somewhat complex character—a quartic curve, for instance—might fit the observations.

† The conception can be illustrated by the march past of troops at a review; the men in the various regiments would have degrees of efficiency depending on their regimental commanders. Thus in the line of march past the saluting-point efficiency would be contingent on distance; but if the troops were drawn up in this line with a random distribution of regiments, there would be no continuous relation between efficiency and distance, *i.e.* the coefficient of correlation would be zero.

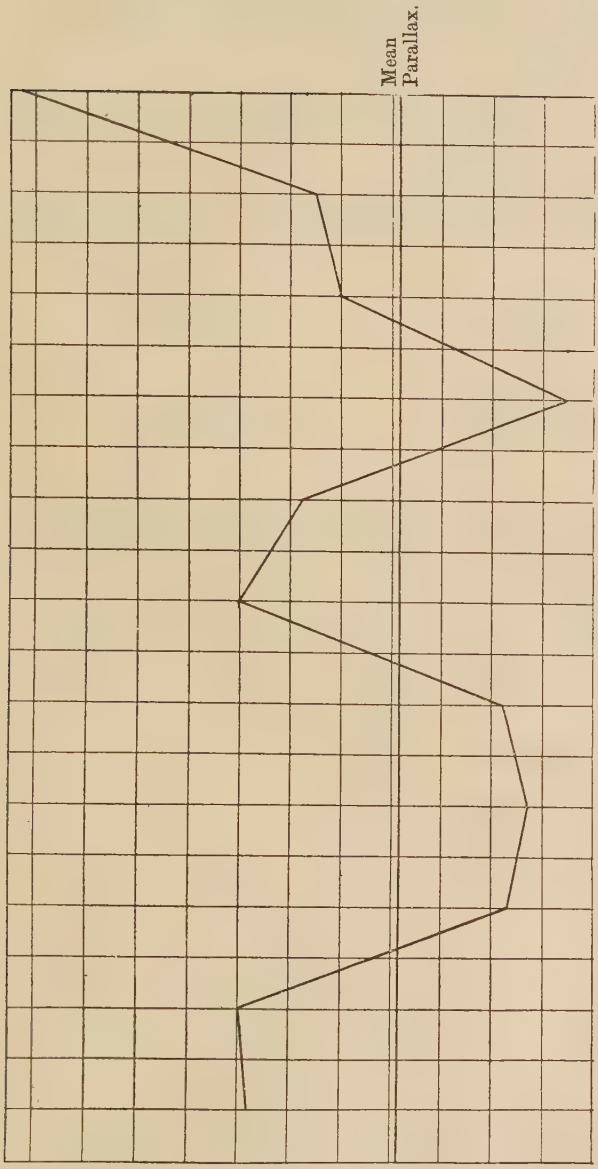

FIG. 1.—Parallax and Stella Magnitude.

sponding distance. It is needless to say that the suggestion of a non-graduated discontinuous relation between luminosity and distance can only be verified or disproved when far more parallax data are available.

(3) The next stage in the work was to inquire whether magnitude was more markedly correlated with any other character than parallax. Colour suggested itself as a character for which definite data were available.

In vol. ix. of the *Annals of the Cape Observatory* will be found a catalogue from which the colour of 159 stars can be extracted. The following colour classification is used at the Cape :—

0 White.
1 Yellowish.
2 Yellow.
3 Deep yellow.
4 Orange yellow.
5 Orange.
6 Orange red.
7 Reddish.
8 Red.
9 Very red.
10 Deepest red.

For the purpose of forming a contingency table from comparatively scanty data, 1, 2, and 3 were thrown together as *yellow*; 4, 5, 6, and 7 were still retained as independent groups; and 8 and 9 thrown together as *red*; 0 and 10 had no frequency in the record.

Table III. gives the correlation table for *visual* magnitude, Table IV. throws it into the form of a contingency table, and Table V. is a contingency table for photographic magnitudes.

TABLE III.

Association of Colour and Visual Magnitude.

Cape Colour Scale.	Magnitude. 4	4·5	5	5·5	6	6·5	7	7·5	8	8·5	9	9·5	Totals.
1	...	...	...	1	...	...	...	...	...	...	...	...	1
2	...	...	...	...	...	1	1	1	...	...	...	...	3
3	...	...	...	...	...	...	·5	1	2	...	3	...	6·5
4	...	...	...	3	2	2	2·5	2	7	2	4	...	24·5
5	...	...	1	1	1	...	5·5	6	4	6	4	2	30·5
6	·5	...	2	1	3	3·5	6	4·5	8	10·5	8	...	47
7	·5	...	...	1·5	·5	2	1	3·5	3	9·5	6	...	27·5
8	...	...	1	·5	1	1·5	1·5	1·5	1·5	·5	3	1·5	13·5
9	...	...	...	...	·5	...	...	1·5	·5	·5	1	1·5	5·5
Totals	1	0	4	8	8	10	18	21	26	29	29	5	159

TABLE IV.

Contingency Table of Colour and Visual Magnitude.

Colour Scale.	Magnitude. 4	5	6	7	8	9	Totals.
Yellow	0	1	1	3·5	2	3	10·5
Orange Yellow	0	3	4	4·5	9	4	24·5
Orange	0	2	1	11·5	10	6	30·5
Orange Red	·5	3	6·5	10·5	18·5	8	47
Reddish	·5	1·5	2·5	4·5	12·5	6	27·5
Red	0	1·5	3	4·5	3	7	19
Totals ...	1	12	18	39	55	34	159

TABLE V.

Contingency Table of Colour and Photographic Magnitude.

Colour Scale.	Magnitude. 6	7	8	9	10	Totals.
Yellow	1	1·5	4	3	1	10·5
Orange yellow	1	5·5	7·5	10·5	...	24·5
Orange	...	4·25	11·25	12	2	29·5
Orange red	·5	7·25	17·25	19·25	2·75	47
Reddish	·5	2·75	5·75	11·75	4·75	25·5
Red	...	2·75	2·75	7·5	1	14
Totals	3	24	48·5	64	11·5	151

The results given by these contingency tables are as follows :

Mean visual magnitude ... ... = $7·72 \pm ·063$

Standard deviation of visual magnitude ... = $1·174 \pm ·044$

Coefficient of contingency between colour and visual magnitude ... ... = ·298

Mean photographic magnitude ... ... = $8·58 \pm ·046$

Standard deviation of photographic magnitude = $·835 \pm ·032$

Coefficient of contingency between colour and photographic magnitude ... ... = ·304

This amount of agreement is extremely satisfactory considering the very considerable difference in the distribution of magnitudes in the two cases. As we possess no quantitative colour scale it is impossible to compare these results with those obtained from straightforward correlation calculations, but it

seems worth while to examine the mean visual magnitude corresponding to each colour group.

Mean magnitude for	all colours	... ...	= 7·92
,,	Yellow stars	... ...	= 7·87
,,	Orange yellow stars	... ...	= 7·67
,,	Orange stars	... ...	= 8·00
,,	Orange red stars	... ...	= 7·84
,,	Reddish stars	... ...	= 8·14
,,	Red stars	... ...	= 8·02

These results, while still very irregular, do show (fig. 2) a general tendency to higher magnitude as we pass from the yellow to the red stars; but it must be remembered that the absence of any quantitative colour scale only allows of a very rough *graphical* appreciation of the relationship.

We see, however, that while the contingency coefficient of magnitude on colour is somewhat less than in the case of magnitude and parallax, there yet appears to be a possibility of a more continuous and graduated relationship in the former case than in the latter. To sum up, then, we find that there is no well-marked and very definite relationship between magnitude and either parallax or colour. Such association as appears to exist from the data at present available seems, however, to point to *quality* of light rather than distance as having a continuous and proportional effect on magnitude.

(4) From what precedes it will be clear that, frequencies being given in terms of magnitude, there is small hope of our being able to approach stellar frequency from the standpoint of a chance distribution, the variate being either distance or colour, the correlations between these and magnitude being far too small to allow of any replacement of magnitude by these variates. If any formula like (iii.) be taken it will be found to give far too high frequencies for stars of the fourth and higher magnitudes.

The only variate which appeared to present itself as a character which might possibly have a chance distribution was the amount of light as determined by equation (i.), p. 446.

We have seen that the amount of light is not closely associated with distance; it is therefore *à priori* conceivable that it is a character of the individual star which, like stature or head-measurements in a population of men, is closely described by a chance distribution of frequency. The main difference would be that in the case of the anthropometric measurements we are able to observe the whole frequency distribution of a random sample, while in the case of the stars we can only observe the frequency of the tail of the distribution—*i.e.* the frequency of those stars whose luminosity is sufficiently great to allow of a count.*

* A somewhat similar case of a knowledge of the "tail" only arises in the case of speed of trotting horses, the record only including the fastest trotters. See *Biometrika*, vol. ii. pp. 2–6.

Accordingly, the first stage of the second part of the inquiry was to investigate whether the counts of stars up to the sixth magnitude could possibly be represented by the tail of a chance

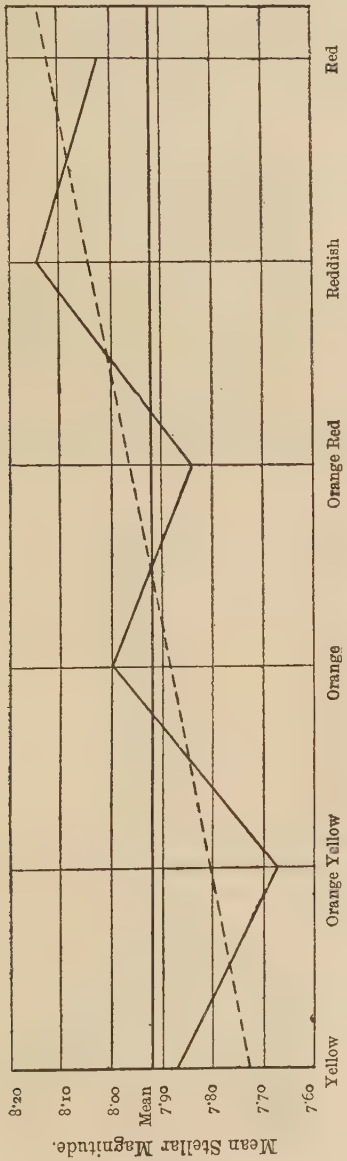

Fig. 2.—Relation between Stellar Magnitude and Colour

frequency curve, beginning in the first place with the ordinary Gaussian or normal curve.

Taking as unit of light the light of a star of the tenth magnitude, the amounts of light corresponding to stars ·75, 1·25, 1·75, 2·25 . . . &c., magnitudes were calculated from (i.). Then if n_1, $n_{1\cdot5}$, n_2, $n_{2\cdot5}$, &c., be the number of stars found from the counts between magnitudes ·75 and 1·25, 1·25 and 1·75, 1·75 and 2·25, 2·25 and 2·75, &c., the corresponding ordinates of the frequency distribution were taken as y_1, $y_{1\cdot5}$, y_2, $y_{2\cdot5}$, &c., where

$$\begin{aligned} y_1\ (x_{0\cdot75}-x_{1\cdot25}) &= n_1 \\ y_{1\cdot5}(x_{1\cdot25}-x_{1\cdot75}) &= n_{1\cdot5} \\ y_2\ (x_{1\cdot75}-x_{2\cdot25}) &= n_2 \\ y_{2\cdot5}(x_{2\cdot25}-x_{2\cdot75}) &= n_{2\cdot5}\ \&c. \end{aligned}$$

the abscissæ being taken as $\frac{1}{2}(x_{0\cdot75}+x_{1\cdot25})$, $\frac{1}{2}(x_{1\cdot25}+x_{1\cdot75})$, $\frac{1}{2}(x_{1\cdot75}+x_{2\cdot25})$, $\frac{1}{2}(x_{2\cdot25}+x_{2\cdot75})$, &c., where x_m represents the amount of light corresponding to magnitude m. Eight values were selected to work upon, namely, y_2, $y_{2\cdot5}$, y_3, $y_{3\cdot5}$, y_4, $y_{4\cdot5}$, y_5, $y_{5\cdot5}$, as being values corresponding to closely known star counts. When this work was begun Pickering's recent paper had not been published, and accordingly Newcomb's values of the counts were used.* These give :—

TABLE VI.

Magnitude.	Frequency.	Ordinate y.	Abscissa x.
2·0	32	·043458	1627·0950
2·5	41	·088248	1026·6290
3·0	78	·266084	647·7575
3·5	135	·730285	408·7075
4·0	240	2·056527	257·8770
4·5	462	6·274310	162·7095
5·0	876	18·855010	102·6630
5·5	1468	50·098461	64·7760

A curve of the form $Y = A+Bx+Cx^2$, where $Y = \log y$ was fitted by the method of least squares to the above eight points. The resulting equation was

$$Y = 1\cdot66915715-\cdot00469533x+\cdot00000177x^2$$

If we throw this into the form of a Gaussian or normal curve

$$y = y_0 e^{-\lambda(x-h)^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(vi.)}$$

we find

$$\left.\begin{aligned} \lambda &= -\cdot00000408,\ h = 1326\cdot36525 \\ y_0 &= \cdot03592 \end{aligned}\right\} \quad \ldots \quad \text{(vii.)}$$

* *The Stars*, p. 53.

Since λ is negative, the index of e in (vi.) is positive, and the values of y will increase with those of x, whether x be positive or negative, from the point at which $x = h$. We see, therefore, that the standard deviation is imaginary, and the Gaussian curve has become a U-shaped curve with a minimum frequency at 1326 light units—*i.e.* between magnitudes 2 and 2·5, and with a frequency rising towards infinity as we approach zero amounts of light. Clearly the portion of the curve for amounts of light greater than correspond to magnitude 2 is quite meaningless. Further the fit between magnitudes 3·5 and 5·5 is a very poor one—it is hardly necessary to give the calculated ordinates. Accordingly we must conclude that *the distribution of light intensities of the stars has not the least approximation to the tail of a chance distribution of the Gaussian type.*

The curve given by y and x in Table VI. has far more the character of an hyperbola with vertical asymptote than any type of frequency curve familiar to us. Accordingly the next stage in the work was to assume

$$y = Cx^{-n} \quad \ldots \quad \ldots \quad \ldots \quad \text{(viii.)}$$

a type of curve which makes the ordinate infinite with zero light, and endeavour to get a better description of the results.

Using the same eight points of Table VI. and applying the method of least squares, we find

$$C = 557185{\cdot}77, \; n = 2{\cdot}2403.$$

Since n comes out positive and greater than unity, it follows at once that on the assumption made a curve of this kind leads to the number of stars in the universe being infinite.

If N_x be the number of stars of light more intense than x, then

$$N_x = \int_x^\infty y dx = \frac{C}{n-1} \cdot \frac{1}{x^{n-1}}$$

$$= 449234{\cdot}67/x^{1{\cdot}2403} \ldots \quad \ldots \quad \ldots \quad \text{(ix.)}$$

Converting light into magnitude by means of the relation

$$x = 10000/(2{\cdot}51189)^m$$

we find

$$N_m = 4{\cdot}91217 \times 10^{\cdot 49612m} \quad \ldots \quad \ldots \quad \text{(x.)}$$

This gives

$$\log N_m = {\cdot}49612m + {\cdot}69127$$

The agreement between the calculated and observed numbers of stars found by calculating the mid-ordinates and then multiplying by the light ranges is moderately good. It is less good if the areas are worked from (ix.), because we have fitted to mid-ordinates and not to areas. If we fit the area curve (ix.) directly to Newcomb's results by least squares we find

$$\log N_m = {\cdot}5296 \times m + {\cdot}5008 \quad \ldots \quad \ldots \quad \text{(xi.)}$$

Using stars to the fifth magnitude from this equation the following results were obtained :—

TABLE VII.

Number of stars of magnitude of less than	1·75	2·25	2·75	3·25	3·75	4·25	4·75	5·25	5·75	6·25	6·75	7·25
Observed after Newcomb ...	23	55	96	174	309	549	1011	1887	3355	5333?	...	...
Calculated ...	27	49	91	166	307	564	1038	1911	3516	6468	11,902	21,880

Here, again, the frequency increases too rapidly for the higher magnitudes, but the agreement is not a very bad one, and it indicates that a light frequency curve of type (viii.) is likely to represent approximately the actual relation between magnitude and frequency for the first part of the distribution. Since the curve rises too rapidly for high magnitudes, it seems to demand that the origin, and accordingly the asymptote, should not be taken at zero light. I assumed, therefore, that the curve was of the form

$$y = C/(x+b)^n \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \text{(xii.)}$$

This curve at once suggests the possibility of determining whether the hypothesis of a limited or an unlimited number of stars best corresponds with the counts of stars of low magnitude. All extrapolation is, of course, only the vaguest suggestion, but the point seemed worth pursuing.

Unfortunately if we put (xii.) in the form

$$\log y = \log C + n \log (x+b) \quad \ldots \quad \ldots \quad \text{(xiii.)}$$

we see that the method of least squares does not lend itself to a ready determination of the constants C, n, and b.

I began by putting for C and n the values already found, and then determining by trial and error an approximate value for b. It was easy to see that the alteration in the origin would considerably better matters. Or the frequency of star-counts of the low magnitudes suggests that the number of stars in the universe may be finite. The next step in the work was to give the approximate values of the constants small variations and determine these by the method of least squares. But at this stage of the work Professor Pickering's recent important paper on the Distribution of the Stars * reached me. This not only provided much matter for consideration, but also the most recent star-counts, these being far more ample and accurate than those on which I had hitherto been working. It seemed, therefore, desirable to discuss the question of bettering the frequency fit by adopting (xii.) and calculating the constants on the basis of Professor Pickering's data.

A word must here be said of Professsr Pickering's own treat

* *Annals of Harvard College Observatory*, vol. xlviii. pp. 149–185.

ment of stellar frequency. He starts from an equation corresponding to my (iii.) and then modifies the constants B and A in the equation $\log N_m = B\ m + A$ to get a better fit. B would = ·60 on the hypothesis of p. 447. Professor Pickering finds

$$\log N_m = \cdot 51m + \cdot 618 \quad \ldots \quad \ldots \quad \text{(xiv.)}$$

It will be noted that the value given above in (xi.) for N_m

$$\log N_m = \cdot 530\ m + \cdot 501$$

based on Newcomb's numbers does not—considering how widely these differ from Pickering's counts—diverge very considerably from Pickering's equation.

Starting from Pickering's counts the next stage in the work was first to better his constants by a rather more general method of curve-fitting, and then, on the basis of this first fit, to endeavour to improve matters by introducing a constant corresponding to the b of my equation (xii.). As in the previous investigation the relation between frequency and amount of light x was assumed to be of the form

$$y = Cx^{-n}$$

whence $N_x = \dfrac{C}{n-1} \Big/ x^{n-1}$ ... ... (xv.)

gives the number of stars with light greater than x.

I now fitted this curve by the method of least squares to the first fourteen entries in Pickering's Table XIV.,* that is, dealing with stars up to magnitude 6·5. The method of fitting was as follows :

$$\log N_x = \nu = \log C - \log(n-1) - \overline{n-1} \log x$$
$$= c - \log(n-1) - (n-1)\,\xi, \text{ say,}$$

we have therefore to make

$$S_1^{14} \{\nu - c + \log(n-1) + (n-1)\xi\}^2 \text{ a minimum.}$$

The type equations of least squares are

$$S(\nu) - 14c + 14 \log(n-1) + (n-1)S(\xi) = 0$$

$$S\{\nu - c + \log(n-1) + (n+1)\xi\}\left(\frac{1}{n-1} + \xi\right) = 0$$

or, using the previous equation,

$$S(\nu\xi) - cS(\xi) + \log(n-1)S(\xi) + (n-1)S(\xi^2) = 0$$

This gives us

$$S(\nu)S(\xi) - 14S(\nu\xi) = \overline{n-1}\,\{14S(\xi^2) - S(\xi)S(\xi)\}$$

which determines $n-1$, and then c is given by the first equation.

* *Loc. cit.* p. 168

Now $\xi = \log x = 4 - \cdot 4m$ * determined ξ; $\nu =$ logarithm of Pickering's counts, and the remainder is only arithmetical calculation. There resulted :

$$n = 2\cdot 27737$$
$$C = 665732\cdot 88$$

or the relation between luminosity and stellar frequency is

$$y = 665732\cdot 88 x^{-2\cdot 27737} \quad \ldots \quad \ldots \quad \text{(xvi.)}$$

i.e. the *area* of this curve up to a given value of x from ∞ represents the total number of stars with a luminosity greater than x. Converting light into magnitude in order to compare with Pickering's equation we find

$$\log N_m = \cdot 51096m + \cdot 60737 \ldots \quad \ldots \quad \text{(xvii.)}$$

There is thus a modification of about 2 per cent. in the constant term.

The following table gives a comparison of the numbers obtained from the above formula and Pickering's numbers on the basis of Pickering's table (XIV.).

TABLE VIII.

Calculated and Observed Stellar Frequencies.

Up to Magnitude.	Observed No. of Stars.	Pickering's Formula.		Present Formula.	
		Total.†	Residual.	Total.	Residual.
·25	5	6	+ 1	5	0
·75	10	10	0	10	0
1·25	18	18	0	18	0
1·75	32	32	0	32	0
2·25	58	58	0	57	− 1
2·75	105	105	0	103	− 2
3·25	193	189	− 4	185	− 8
3·75	336	339	+ 3	334	− 2
4·25	589	610	+ 21	601	+ 12
4·75	1067	1098	+ 31	1083	+ 16
5·25	1972	1975	+ 3	1949	− 23
5·75	3562	3552	− 10	3511	− 51
6·25	6284	6390	+ 106	6322	+ 38
6·75	11004	11495	+ 491	11385	+ 381
7·25	17955	20654	+2699	20505	+2550
Mean square root residual up to and including magnitude 6·75			135	...	103

* See p. 450, $x = l = \dfrac{10000}{(2\cdot 51189)^m}$

† I differ from a considerable number of Pickering's calculated values, supposing him to have determined them from (xiv.) above. See his p. 167.

I think, therefore, that we may conclude that a somewhat better result than Pickering's, but of the same form, can be obtained to represent his counts. But this result still indicates the point noted above, *i.e.* that a formula of type (viii.) gives too great a frequency after we have passed the limit of the lucid stars. Both forms of the equation give results diverging fairly rapidly from observed numbers after magnitude 6 has been passed.

Using the improved form of Pickering's formula, I now turned to the problem of substituting equation (xii.) for (viii.). Recognising the fact that b can only mean a small modification in x, I retained at first the same values of C and n as I had found before, and gave b the series of values recorded in Table IX.

TABLE IX.

Effect of Limiting the Number of Luminous Stars.

Value of b.	Mean Square Root Residual.	Total Number of Stars.
0	103	Infinite.
·1	83	9,870,820
·2	67	4,072,178
·3	51	2,426,012
·4	40	1,679,964
·5	40+	1,263,310

We see from this table that a very slight alteration in the value of b produces an enormous difference in the total number of stars. While the agreement between observed and calculated frequencies, as shown by the mean square-root residual, improves steadily as we raise b from 0 to ·4, the total number of stars diminishes from infinity to 1,679,964, a result obviously far too small.

Or if we use the formula

$$y = 665732{\cdot}88(x+{\cdot}4)^{-2{\cdot}27737} \quad \ldots \quad \ldots \quad \text{(xviii.)}$$

to connect frequency $y\delta x$ with luminosity x, we shall have a result for the number of lucid * stars more than three times as good as Pickering's † equation ; but this improvement is at the cost of limiting the total number of luminous stars to about a million and a half.

Taking Pickering's counts against the numbers obtained from (xviii.) we have Table X. The agreement is certainly noteworthy, considering that we are fitting a curve with only three constants to fourteen points, or rather areas.

* Including under the term "lucid" all stars less than 6·75 in magnitude as being possibly visible to some naked eyes.

† Mean square residual less than 40. Pickering's numbers give 135.

TABLE X.

Distribution of Stars.

Up to Magnitude.	Observed Number.	Calculated Number.	Difference.
·25	5	5	0
·75	10	10	0
1·25	18	18	0
1·75	32	32	0
2·25	58	57	− 1
2·75	105	103	− 2
3·25	193	185	− 8
3·75	336	333	− 3
4·25	589	600	+ 11
4·75	1067	1078	+ 11
5·25	1972	1937	− 35
5·75	3562	3476	− 86
6·25	6284	6222	− 62
6·75	11004	11102	+ 98
7·25	17955	19703	+1748

I then proceeded to modify the C and n constants on the assumption that $b = \cdot 4$ is approximately the correct value. I found $y = 667026/(x+\cdot 4)^{2\cdot 27692}$, or

$$\log N_x = 5\cdot 7179792 - 1\cdot 27692 \log (x+\cdot 4) \quad \ldots \quad \text{(xix.)}$$

TABLE XI.

Allowance for Periodic Variation from Simple Law.

Up to Magnitude.	Value of n from Count.	Observed Number of Stars.	Calculated Value from (xix.).	Δ (xix.).	Calculated Value from (xx.).	Δ (xx.).
·25	1·28691	5	5	0	5·5	+ ·5
·75	1·27511	10	10	0	10	0
1·25	1·27503	18	18	0	18	0
1·75	1·27658	32	32	0	31	− 1
2·25	1·27561	58	57	− 1	56	− 2
2·75	1·27466	105	103	− 2	105	0
3·25	1·27110	193	186	− 7	192	− 1
3·75	1·27637	336	335	− 1	338	+ 2
4·25	1·28119	589	602	+ 13	589	0
4·75	1·28002	1067	1083	+ 16	1062	− 5
5·25	1·27314	1972	1946	− 26	1959	−13
5·75	1·27170	3562	3490	− 72	3560	− 2
6·25	1·27518	6284	6246	− 38	6281	− 3
6·75	1·28105	11004	11142	+138	11003	− 1
Mean square-root residual		...	...	44*	...	3·9

* This value is rather larger than that given by (xviii.), but the formula gives a considerably better result up to magnitude 6·25.

In Table XI. we have (*a*) the values given by this formula, (*b*) the values of the constant $(n-1)$ of the logarithmic term, which would in each case give the observed counts. If these values of n be plotted to the magnitudes, a periodic deviation from the mean value of n at once becomes visible. The period is nearly 2·5 magnitudes with maxima at 2, 4·5, 7 magnitudes, and *roughly* an amplitude of about ·00510.

We may accordingly take

$$\left.\begin{aligned} \log N_x &= 5{\cdot}7179792 - n_m \log (x+{\cdot}4) \\ \text{where} \quad n_m &= 1{\cdot}27692 + {\cdot}00510 \cos 2\pi\left(\frac{m+{\cdot}5}{2{\cdot}5}\right) \end{aligned}\right\} \quad \dots \quad \text{(xx.)}$$

No doubt an even better fit could be obtained with a little trouble, but the mean square-root residual is so small that a closer approximation in the present state of the counts is hardly worth attempting.

Hence I think we may conclude that, confining our attention to the lucid stars (*i.e.* $m<6{\cdot}75$), the frequency distribution of magnitudes is more closely reached on the assumption that the number of stars belonging to this group is limited. Further, there appear to be periodic deviations from a formula of the Pickering type, allowance for which enables us to reduce his mean square-root residual from 135 to 3·9. The latter residual is probably well within the error of the counts.

We have seen that, while we confine our attention to the lucid stars, there is no sensible correlation between magnitude and distance by which distance increases in any way proportionately with magnitude. At the same time, there appear to exist sensible deviations from the mean parallax which are beyond the errors of random sampling. These deviations (fig. 1) have, roughly, a period of about five magnitudes from maximum parallax to maximum parallax. This is about double the period noted in the frequency of the lucid stars. Thus we have the suggestion, even if it be only of the vaguest kind, that the bulk of the lucid stars may belong to a separate universe within which magnitude is not mainly determined by parallax or distance, but is more closely associated with colour, and thus probably with chemical or physical condition. It is suggested that the relation of magnitude to spectroscopic class may be far more important than that of magnitude to distance, and I propose in the near future to form contingency tables of magnitude and spectroscopic classes.

5. *General Summary.*

(*a*) Any character of a star which may be supposed to follow a chance distribution of frequency would enable us, if the "tail" of the distribution were known, to form some estimate of the total number of stars.

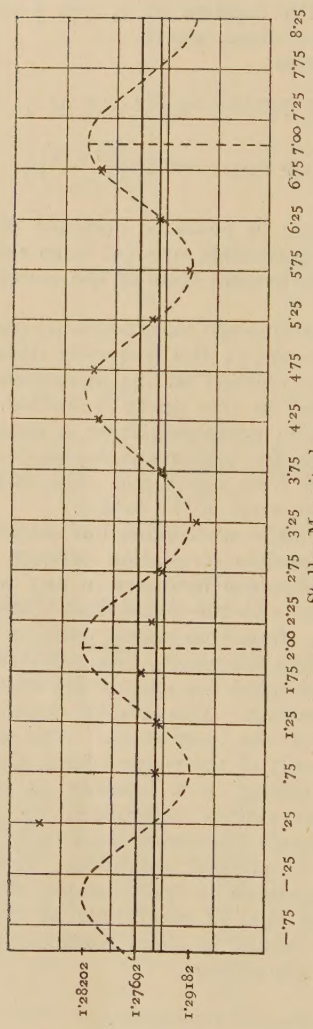

Fig. 3.—Periodicity in Value of n_m.

N.B.—No attempt must be made to judge the accuracy of fit of the periodic curve beyond the limits of magnitude of 2·25 to 7·00. Below 2·25 the whole amplitude of n does not denote the alteration in the frequency by a single star. The correspondence between 2·5 and 7·0 is that upon which weight is here placed. Pickering's constant value of n^m is 1·275.

(*b*) The only character which appears to be at present available is magnitude ; but magnitude is a physiological rather than a physical character, and the chance distribution must only be applied to quantities of the latter kind.

(*c*) An investigation was first made of the relation between magnitude and distance. So far as the lucid stars are concerned there appears, on the basis of parallaxes at present determined, to be no decrease of parallax with increase of magnitude. There is no sensible correlation between the two. A sensible relation was found between parallax and proper motion sufficient to indicate that parallax is really a measure of relative distance, but the relation was not intense enough to enable any useful prediction of parallax to be made from a knowledge of proper motion. On the other hand, both visual and photographic magnitudes were found to be sensibly associated with colour. Investigating further the relation between parallax and magnitude, it seems probable that definite variations of parallax are associated with magnitude, and are possibly of a periodic character ; but a continuous change of one character with the other is not appreciable—at any rate on the present data—so far as the lucid stars are concerned.

(*d*) Taking light as the variable, it was found that no curve approaching the Gaussian type could possibly give the frequency of stars for various quantities of light. The general type of the distribution was seen to be of the form

$$y = \mathrm{C}/(x+b)^n$$

If we put $b = 0$, this gives us, when we replace light by magnitude and y by $\mathrm{N}_x = \int_x^\infty y\,dx$, the form

$$\log \mathrm{N}_x = \mathrm{C}' + n'm$$

where m is the magnitude and C' and n' are constants. This form may be directly deduced by the assumptions—

(1) That Fechner's law in *its simplest form* holds ;

(2) That magnitude varies inversely as the square of the distance ;

(3) That the stars are scattered uniformly through space.

All these hypotheses are almost certainly erroneous, yet it is remarkable how closely their combined errors give the true result. Pickering has already considered the application of a formula of the above kind, and shown that it still approximates to the observed frequencies.

Pickering's formula can be made to give better results, in the ratio of 135 to 103 (testing by the residuals), by choosing somewhat different constants. It can be improved in the ratio of 135 to 40 by supposing b is not zero, or that the number of stars of the system to which the lucid stars in the main belong is not infinite. It is then seen that the deviations in the

frequency from the proposed formula are fairly closely periodic, the period being 2·5 magnitudes. In this manner the calculated frequencies up to magnitude 6·75 can be obtained with a mean square-root residual of less than four stars. Greater exactitude can hardly be hoped for until the probable error of the counts is far less than at present.

(*e*) The general conclusion is reached that magnitude is not, so far as the lucid stars are concerned, mainly, if at all, a question of distance. The arrangement of these stars differs from a pure random distribution throughout space, based upon magnitude as a function of distance, by a periodic variation in frequency which appears somewhat similar, although of only half the period, to that which occurs in the relation of parallax to magnitude. Further, a better fit is obtained for the frequencies of the lucid stars if we assume the asymptote to the frequency curve not to be at zero light. This suggests that the system to which the lucid stars belong may possibly be a limited system of a definite and not random structure.

This work was undertaken in the Drapers' Company Statistical Laboratory at University College, London, and I have to thank the staff there for suggestions and assistance during the progress of the work, in particular Mr. J. Blakeman, M.Sc., for aid in the very laborious calculations of which only the results appear in this paper.

Errata in Monthly Notices, vol. lxiv. No. 6.

Page 554, 1906, Arguments vi. and vii. should be transposed.
„ 557, 1906, B *should read* 35° 48′ 48″·12.
„ „ 1907, log *c* „ „ 9·5734945.

Erratum in Annual Report, 1906.

In the Council Note on Groombridge's Catalogue the following should be inserted after "and probably a few others" (p. 231, line 18): Hornsby's observations would take an honourable place in the list.